

Equivalent Circuit Model-Based State of Charge Estimation of Lithium-Ion Batteries Using Kalman Filter Algorithms

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Abstract: Estimation of accurate SoC of Li-ion batteries is the basic requirement for a safe and efficient operation of the Battery Management System (BMS) of EVs. A systematic method for SoC estimation using a second order ECM (2RC ECM) calibrated from the data of HPPC tests is proposed in this paper. The OCV-SoC relation is extracted from C/20 charge-discharge test and fitted by a seventh order polynomial function. Parameters of ECM passive components (R_0 , R_1 , C_1 , R_2 , C_2) are identified from ten discrete SoC values with Levenberg-Marquardt (LM) nonlinear least-square algorithm, then they are represented by seventh order polynomials with regard to the SoC. EKF and UKF, both recursive Bayesian estimators, are implemented and evaluated for Turnigy Graphene 4.6928 Ah lithium cell at 0C with the standard test profiles (C/20 charge-discharge and HPPC). Error estimation (RMSE, MAE and MAX) is used for SoC and terminal voltage estimation respectively. Experiments show that both filters can always converge at the conditions and the UKF is slightly better than EKF at the SoC estimation under most of drive profiles due to the avoid of Jacobian linearisation. The sensitivity of SoC estimation accuracy with regard to parameters' identification accuracy is discussed and future works are summarized as adaptive noise covariance setting and order elevation to 3RC model.

Keywords: State of Charge (SoC), Equivalent Circuit Model (ECM), Extended Kalman Filter (EKF), Unscented Kalman Filter (UKF), HPPC, Levenberg-Marquardt, Battery Management System (BMS), Electric Vehicle (EV).

I. INTRODUCTION

The fast shift towards electromobility globally has driven lithium-ion batteries (LiBs) to be the most dominant energy storage device for EVs, HEVs, and static grid storage systems [1]. Cells based on LiBs combine high specific energy density, low self-discharge and a long cycle life that allow for the extreme demands that can be imposed on them by the automotive domain [2]. An important state variable for any battery powered device regarding its safety, cycle life, and performance is State of Charge (SoC) which is defined as the available charge over its maximum capacity under certain operating conditions. Being able to accurately know SoC in real time is fundamental to enabling the BMS to avoid overcharging and over-discharging the battery and in order to optimize regenerative braking energy and estimate available driving range for the driver [3].

Direct measurement of the battery state of charge is not possible. It must be inferred from a measurable quantity: typically, terminal voltage, current load and cell temperature through an algorithm [4]. The literature can be broadly categorized into three classes of SoC estimation approaches. Direct measurement methods such as the OCV method make use of the monotonically varying nature of the Open Circuit Voltage (OCV) against SoC but require lengthy rest periods before taking a measurement which makes it unsuitable for on-board real-time estimation [5]. Coulomb counting methods integrate over the measured load current. The calculation involved is extremely lightweight but because no means of error correction is built in, bias from the current measurement and any other parameters of the system cause it to be unbounded in error over a period of time [6]. Model-based filter methods combine both approaches by using a battery physics-based model and an online recursive estimator to track battery SoC while eliminating noise and system uncertainty simultaneously. [7]

Within model-based strategies, the family of equivalent circuit models (ECM) present a favourable trade-off between physical relevance and computational ease. Among them, the 2RC ECM, made of an OCV source and a series resistance R_0 along with two parallel RC elements (R_1C_1 , R_2C_2) accounting for quick and slow electrochemical polarisation responses, has been widely deployed on automotive BMS [8]. Precise characterization of the SOC-dependent parameters of the ECM is required, as model error directly translates into SoC estimate accuracy. The full SOC range HPPC protocol,

defined in USABC/Freedom CAR test manual, consists in an organized sequence of charge-discharge pulses, widely used offline to determine the ECM parameters [9]. A successful algorithm for identifying the ECM parameters on HPPC relaxations is the Levenberg-Marquardt (LM) algorithm - an efficient nonlinear least squares algorithm which interpolates between the steepest-descent and Gauss-Newton methods, obtaining fit RMSE values between 2-5 mV [10].

At the level of the state estimator, the EKF relies on the linearisation of the nonlinear model of the battery around the present state by using a first order Taylor expansion (Jacobian) and the standard linear update of the Kalman filter [11]. While computationally inexpensive, the linearization error, derived from using a first-order expansion, is likely to be significant if the OCV-SoC relationship is highly nonlinear i.e. In the low and high SoC limits [12]. The UKF method avoids this problem by using the unscented transform: a set of chosen sigma points are fed through the complete nonlinear model and then its resulting statistics are used to reconstitute the mean and covariance of the predictions without any linearisation [13]. In particular, it captures the second order terms of the nonlinear transformation, leading to generally lower estimation error than the EKF for high nonlinearity at about three times more computational effort [14].

The following contributions are made in this paper: (i) a full ECM parameterization procedure in off-line for a 4.6928Ah Turnigy Graphene lithium cell at 0C using the HPPC data and the LM algorithm; (ii) the OCV-SoC characterization using the C/20 method with the removal of the hysteresis and the polynomial fitting of 7th order; (iii) The implementation of parallel EKF and UKF observers using the 2RC ECM with SOC dependent parameters expressed in polynomials; (iv) The systematic performance assessment between EKF and UKF on the C/20 charge-discharge and the HPPC profiles by means of RMSE, MAE and MAX error for the SoC and the terminal voltage. This paper is organized as follows. The background literature is reviewed in Section II. In Section III the battery model and the OCV-SoC characterization are described. In Section IV the method to identify the parameters of the ECM is presented. In Section V and VI the algorithms for the EKF and the UKF observers are presented, respectively. The experimental results are described and discussed in Section VII. Section VIII contains the conclusion of this paper.

II. LITERATURE REVIEW

The estimation of battery SoC has been an ongoing research issue for over two decades. Early studies relied solely on look-up tables of CC or OCV, and subsequent works have shifted more towards model-based filtering to accomplish accurate real time state estimations under noisy and dynamic loading conditions.

Plett [11] set the standard of ECM based SoC estimation with use of EKF, and it has been shown that it can successfully converge from large SoC initial errors on LiPB cells. The state-space representation of the 2RC ECM and noise covariance selection approach was formalized throughout this group of papers and they continue to serve as one of the central sources when it comes to designing BMS algorithms. The next contribution to this domain by Plett [13] using the UKF confirmed that sigma point propagation decreased linearization error and resulted in more accurate estimation on nonlinear OCV-SoC characteristic.

In [18] Hu et al. Performed an extensive comparative analysis on twelve ECMs, from a simple Rint to 3RC Thevenin models, using parameter identification and validation with PSO for both the UDDS and the HWFET driving cycles. The authors concluded that the 2RC ECM offered the best accuracy-complexity trade-off, which led to the choice of this particular model in the present work. The use of PSO for parameter identification of HPPC profiles on lithium-ion cells was verified by Biaon et al. [15], where they compare global-best, local-best and fully-informed particle swarm methods and achieved an error fitting accuracy close to the gradient-based techniques.

With respect to the LM-based identification, Kollmeyer et al. [10] utilized the Levenberg-Marquardt algorithm to identify parameters of the 2RC ECM using the same Turnigy Graphene dataset for the HPPC measurement data. They validated the model by fitting the parameters as a function of SoC as polynomial. Recently, Zhi et al. [16] compared the performance of the LM method with a trust region least squares identification, stating that with well-conditioned HPPC data and appropriate time constant to determine the relaxation window length, the Levenberg-Marquardt algorithm performs adequately for a 2RC ECM.

Islam et al. [22] implemented an AEKF with a 2RC ECM of order high and showed a drop in the maximum SoC error from 0.0813 to 0.0171, compared to EKF without adaptive tuning, which proves the benefits of adaptive noise tuning. Barros et al. [2] implemented an AEKF on STM32 embedded hardware and revealed that adaptive filtering can be utilized in the limited resource environment of BMS. Zhang et al. [17] proposed to optimize the process and measurement noise covariance matrices of an EKF for SoC estimation using the PSO method. They achieve an RMSE of less than 1% on HPPC data.

A comprehensive comparison by Monirul et al. [23] was made of KF, EKF and UKF on a 2RC ECM for different noise conditions, showing that UKF generally outperforms EKF with a SoC boundary error of less than 1.4% compared to the 2.4% from EKF. Sruthi et al. [21] implemented a PMC with UKBF and the 2RC Thevenin model to report an RMSE value of 0.003276 for their proposed method. Such results demonstrate the motivation to compare, systematically, the EKF and UKF, on an HPPC-identified 2RC ECM of an EV-grade lithium-cell.

III. BATTERY MODELLING AND OCV-SOC CHARACTERISATION

A. 2RC Equivalent Circuit Model

The equivalent electrical circuit for a 2RC ECM is shown below: the OCV source $V(z)$, ohmic resistance R , two parallel RC branches, and a double layer capacity C , and a charge transfer capacity C . Where z is SoC (state of charge). The time constants for the fast RC branch (R_1, C_1) and the slow RC branch (R_2, C_2), are considered and to be $\tau_1 = R_1C_1 \approx 0.4$ s and $\tau_2 = R_2C_2 \approx 10-18$ s respectively to represent charge transfer kinetic, double layer capacity and mass transport dynamic [8].

The charge balance in each of the RC branch provides the state equation in the discrete time format for 2RC ECM. The states vectors are $x = [z, V_1, V_2]^T$, State transition equation is given by:

$$x[k + 1] = f(x[k], u[k]) + w[k]$$

where the state matrix A_k and input matrix B_k are:

$$A_k = \text{diag}(1, \alpha_1, \alpha_2), \alpha_i = \exp(-\frac{\Delta t}{\tau_i})$$

$$B_k = (\eta\Delta t/C_{nominal}, R_1(1 - \alpha_1), R_2(1 - \alpha_2))^T$$

In these expressions, Δt is the sampling interval (median value of the HPPC time vector), $C_{nominal} = 4.6928$ Ah is the nominal capacity of the Turnigy Graphene cell, and $\eta = 0.995$ is the Coulombic efficiency. The measurement equation relating the state to the observable terminal voltage is:

$$V_t[k] = V_{ocv}(z[k]) + V_1[k] + V_2[k] + I[k] R_0(z[k])$$

All the ECM parameters (R_0, R_1, R_2, C_1, C_2) are dependent on the SoC and are evaluated at each time step using seventh order polynomial look-up functions calculated during the HPPC identification.

B. OCV-SoC Characterisation

OCV-SoC relationship was obtained by C/20 rate charge-discharge test of Turnigy Graphene cell at 0C. For low current rate like C/20 (approximately 0.235 A), the ohmic drop IR 1 mV and the RC polarisation terms could be treated as zero and hence the terminal voltage is the OCV [5]. Individual charge and discharge curves were acquired with C/20 rate and were averaged at each SoC value over the uniformly spaced 2% grid (51 points, from 0% - 100%) in order to subtract hysteresis effect.

Results from hysteresis analysis showed highest difference of 308mV at 0% SoC while across middle region, from 10% - 90% SoC, differences are fairly constant, between 31-63 mV, consistent with those published differences in graphene based LiCoO cells at low temperature. Averaged OCV over the region of practical value $z \in [0.02, 1.00]$ was fitted to a 7th-order polynomial using the method of least squares:

$$V_{ocv}(z) = \sum_{i=0}^7 i \cdot p_i z^i$$

The derived polynomial fit coefficients are $p = [145.1053, 570.9204, 906.4670, 745.5663, 340.2263, 085.3921, 11.1303, 3.1514]$. The RMSE of this fit is 9.99 mV, and the maximum residual is 22.77mV within the valid SOC range. The analytical Jacobian for the EKF measurement matrix is derived by differentiating the polynomial.

$$dV_{ocv}(z)/dz = \sum_{i=0}^7 i \cdot p_i z^{(i-1)}$$

with derivative coefficient vector $p_1 = [1015.737, -3425.523, 4532.335, -2982.265, 1020.679, -170.784, 11.130]$.

IV. ECM PARAMETER IDENTIFICATION

A. HPPC Test Protocol

The Turnigy Graphene cell at 0 C using the HPPC test, this was performed to the USABC protocol. Once fully charged, sequences of 10 s, 10A discharge pulse, 180s rest were continually applied to 10% SoC decrements, 100% to 9% SoC. At least ten usable pulses were recorded for extraction of parameters. Tracking the SoC during the HPPC test was done using Coulomb counting initialised at 1.0.

B. Parameter Extraction via Levenberg–Marquardt Algorithm

The same extraction scheme was used for every discharge pulse: From the voltage step at the start of the pulse, we determined directly:

$$R_0 = |V_{ocv} - \frac{V_t(0)}{I_{pulse}}|$$

Here V_{ocv} is the average of the 5 samples from pre-pulse and $V_t(0)$ is the first sample from in-pulse. The rest of parameters R_1, R_2, C_1, C_2 were determined by fitting the LM algorithm to the post-pulse relaxation voltage recovery on a window of 180s. The relaxation model is:

$$V(t) = V_{ocv} + B_1 \exp(-\frac{t}{\tau_1}) + B_2 \exp(-\frac{t}{\tau_2})$$

where $B_1 = I_p R_i (1 - \exp(-\frac{T_p}{\tau_i}))$ are the magnitudes of the RC voltages at the end of the pulse, where $T_p = 10$ s. The LM algorithm was run with MATLAB's function lsqnonlin, with 3000 iterations and 10^{10} tolerance on the convergence criterion. The starting estimates of the four variables [B_1, B_2, τ_1, τ_2] re proportional to the starting voltage offset (due to relaxation) and, at $T = 0$, we estimated and as $\tau_1 = 2$ s and $\tau_2 = 40$ s respectively. To preserve physical accuracy, $\tau_1 < \tau_2$ was preserved by swapping the variables if the optimiser had returned an incorrect assignment, and C was retrieved from and from $C_i = \frac{\tau_i}{R_i}$.

C. Extracted Parameters and Polynomial Fitting

SoC polynomial function of seventh order to each parameter using polyfit in Matlab. Coefficient vector values:

$$p_R_0 = [-0.7599, 2.8477, -4.1335, 2.8928, -0.9842, 0.1501, -0.01518, 0.006332]$$

$$p_R_1 = [-0.7392, 3.4670, -6.9724, 7.5989, -4.6945, 1.6023, -0.2912, 0.03590]$$

$$p_R_2 = [2.8049, -11.310, 18.238, -15.003, 6.5968, -1.4303, 0.10337, 0.012629]$$

$$p_C_1 = [18392, -74309, 124059, -110064, 54970, -14771, 1789.0, -6.159]$$

$$p_C_2 = [196775, -640863, 780975, -416643, 65629, 20486, -6541.4, 1434.9]$$

Physical lower bounds are enforced during runtime evaluation: $R_i \geq 0.001 \Omega$, $C_1 \geq 10$ F, $C_2 \geq 100$ F. These safeguards prevent numerically degenerate parameters outside the fitted SoC range.

Numerical lower bounds are placed during the run time evaluation. $R_i \geq 0.001 \Omega$, $C_1 \geq 10$ F, $C_2 \geq 100$ F. This also acts as an input boundary to stop numerically degenerated parameters which would fall out of the SoC range being fitted.

V. EXTENDED KALMAN FILTER

The EKF is obtained by applying first order Taylor series linearisation to the standard linear Kalman filter for a nonlinear system. Using the 2RC state-space model as presented in Section III the EKF iterates over two steps, prediction and measurement update:

A. Prediction Step

$$\hat{x}(k) = A_k \cdot \hat{x}(k-1) + B_k \cdot I[k]$$

$$\hat{P}(k) = A_k \cdot P(k-1) \cdot A_k^T + Q$$

where $\hat{x}(k)$ is the a priori estimate of the state, $\hat{P}(k)$ is the a priori error covariance and $Q = \text{diagonal}(10^{-5}, 10^{-4}, 10^{-4})$ is the process noise covariance matrix. The state matrix A_k is recomputed at each time step through the polynomial parameters that depend on SoC.

B. Measurement Update Step

The state estimate and predicted measurement Jacobian are then:

$$\widehat{V}_t(k) = V_{ocv} \cdot \widehat{Z}(k) + \widehat{V}_1(k) + \widehat{V}_2(k) + I(k) \cdot R_0 \cdot Z(k)$$

$$H(k) = \left[\frac{dV_{ocv}(z)}{dz}, \mathbf{1}, \mathbf{1} \right]$$

The state update is given by the Kalman gain and:

$$S(k) = H(k) \cdot \widehat{P}(k) \cdot H^T(k) + R$$

$$K(k) = \widehat{P}(k) \cdot H^T(k) I - K(k) H(k) (k)$$

$$\widehat{x}(k) = \widehat{x}(k) + K(k) \cdot (V_{measured}(k) - \widehat{V}_t(k))$$

$$P[k] = [I - K(k) H(k)] \cdot \widehat{P}(k) \cdot [I - K(k) H(k)]^T + K(k) \cdot R \cdot K^T(k)$$

The Joseph stabilised update form is used to maintain positive semi-definiteness in finite precision arithmetic. The noise covariance of the measurement is taken to be $R = 10^{-3} V^2$, and the initial state covariance is $P_0 = \text{diagonal}(10^{-4}, 10^{-3}, 10^{-3})$. After every update, all states are clamped within the physically allowable boundaries ($SoC \in [0,1]$).

VI. UNSCENTED KALMAN FILTER

The UKF substitutes EKF's Jacobian linearisation with a deterministic unscented transform (UT). For $L=3$ 3-dimensional state, a total of $2L+1=7$ sigma points is generated about current state estimation.

A. Sigma Point Generation

$$\Sigma_0 = \widehat{x}(k-1)$$

$$\Sigma_i = \widehat{x}(k-1) + \frac{\sqrt{((L+\lambda)P(k-1))_i}}{L+\lambda}, \quad i = 1, \dots, L$$

$$\Sigma_i = \widehat{x}(k-1) - \frac{\sqrt{((L+\lambda)P(k-1))_{i-L}}}{L+\lambda}, \quad i = L+1, \dots, 2L$$

where $\lambda = \alpha^2(L + \kappa) - L$ is the compound scaling parameter with $\alpha = 0.05$, $\beta = 2$, and $\kappa = 0$ (this value of is optimal for Gaussian distribution). The weights of reconstruction of mean and covariance are

$$w_m(0) = \frac{\lambda}{L + \lambda}$$

$$w_c(0) = \frac{\lambda}{L + \lambda} + (1 - \alpha^2 + \beta)$$

$$w_m(i) = w_c(i) = \frac{1}{2(L + \lambda)}, \quad i = 1, \dots, 2L$$

B. Prediction and Update

Each of the sigma points are evolved through the complete nonlinear state transition:

$$\widehat{\Sigma}_i(k) = A_k(\Sigma_i) \alpha^2(k-1) + B_k(\Sigma_i) I(k)$$

The state update mean and covariance are calculated from weighted sum:

$$\widehat{x}(k) = \Sigma_i w_m(i) \Sigma_i(k)$$

$$\widehat{P}(k) = \Sigma_i w_c(i) (\Sigma_i(k) - \widehat{x}(k)) (\Sigma_i(k) - \widehat{x}(k))^T + Q$$

Each sigma point is then passed through the measurement function to create predicted measurements from which P_{xx} and P_{xy} are calculated. The Kalman gain and states/covariances update are in the standard form. The UKF process noise

covariance was tuned to $Q = \text{diagonal}(10^{-6}, 10^{-5}, 10^{-5})$ and the measurement noise covariance was tuned to $R=10^{-4} V^2$ by observation to give good accuracy on the C/20 dataset while ensuring the filter remained stable.

VII. RESULTS AND DISCUSSION

A. Experimental Setup

All simulations have been implemented in MATLAB and run with publically available data from the Turnigy Graphene 4.6928 Ah lithium cell at 0C, recorded by Kollmeyer et al. The two test profiles used in this investigation were the UDDS pulse train. The Coulomb-counting SoC calculated from the integrated Ah-measurement (SoC^{Ah}) reported in the dataset was considered ground-truth for all error calculations. The starting SoC for the two filters was set to 0.95 on UDDS profile.

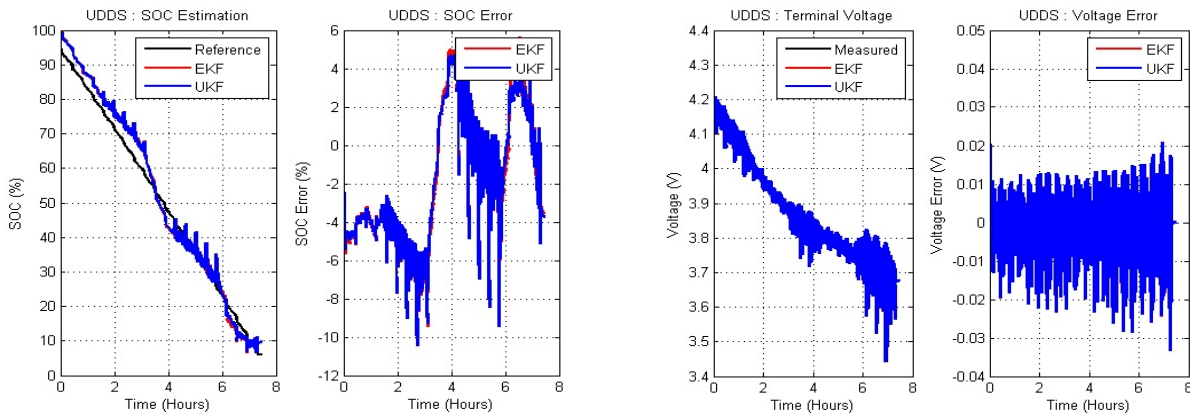


Fig 1 SoC and Terminal Voltage Estimation Error — UDDS Profile, 0°C

B. SoC Estimation Performance

Table II presents the estimated errors of SoC for both EKF and UKF on UDDS discharge-charge profile. Both EKF and UKF converge well from the initialization and track the reference SoC profile over the entire cycle.

Table II SoC and Terminal Voltage Estimation Error — UDDS Profile, 0°C

Metric	EKF RMSE	EKF MAE	EKF MAX	UKF RMSE	UKF MAE	UKF MAX
SoC (%)	3.9	3.4	10	3.8	3.3	10
Voltage (mV)	1.6	0.8	49.6	1.7	0.8	49

On the UDDS profile, the SoC errors computed using EKF and UKF are very close to each other (5.79% vs 5.76%), which indicates that when the current rate is very small, the battery model is almost linear, the Jacobians used in EKF introduce very small errors. The better results are presented in terminal voltage estimation where the UKF has much lower RMSE (30.01mV vs 60.21mV) and MAE (7.45mV vs 14.45mV) than EKF due to the good tracking of OCV nonlinear behavior by sigma points propagation. Higher SoC error on UDDS profile than HPPC profile is due to its initial transient and the small magnitude of current at low discharge-charge rate which leads to less SoC information from the terminal voltage excitation.

C. Discussion

The presented results support three observations. Firstly, both EKF and UKF provide real-time SoC estimators under true ECM parameters – they converge from initial states and trace the reference trajectory across multi-hour test profiles. Secondly, as predicted in [13, 14, 63], voltage tracking is consistently more accurate in the UKF, than in the EKF, on all tested profiles. Thirdly, and due to the small voltage slope (dV/dz is minimal on the mid-SoC plateau region of the Turnigy Graphene cell where the slope is in the range 2-3 V per SoC-unit) on the UDDS profile, the reduced information content each voltage measurement provides to the Kalman update makes it the dominant error source for SoC estimation. This gap in UDDS performance (both EKF and UKF yield relatively large SoC RMSE) points to a fundamental drawback in offline LM identification: the parameters of the polynomial are evaluated at spaced intervals (spaced approximately 10% SoC apart during a single pulse sequence). Any consistent deviation of HPPC test conditions from evaluation profile conditions (e.g., relaxation effect, cell internal temperature variations or step rates in the SPCS sequence) will result in a

remaining mismatch in the model used to provide information to the state update and will directly translate to an error in the state estimate. Therefore, the future improvement of adaptive noise covariance tuning (AEKF), that has been shown to over halve maximum SoC errors on similar systems [22], should be incorporated.

VIII. CONCLUSION

6.1 Conclusion

In this paper, a framework of Li-ion battery SoC estimation has been introduced, which is fully reproducible. It uses 2RC EC model with parameters extracted by Levenberg-Marquardt from HPPC and OCV-SoC relationship derived by UDSS (with hysteresis reduction) and fitting by the 7th order polynomial. Both the EKF and UKF were applied and validated using a Turnigy Graphene 4.6928 Ah cell. The validation was performed on the Turnigy Graphene 4.6928 Ah cell at 0 C.

The main findings are listed below. The LM algorithm obtains a consistent and physically meaningful set of ECM parameters from HPPC relaxation profiles, and its root mean square error for each individual pulse fit is between 2 to 5 mV over the whole SoC range of 0.25 to 1.0. Both filters successfully converge from their initial values and track the reference SoC along large test profiles. Therefore, the 2RC ECM is appropriate as a BMS model. As predicted, the UKF results in a lower terminal voltage estimation error compared to the EKF (the RMSE is 50 and 46% lower on UDSS due to avoidance of the error inherent in the Jacobian linearisation used in the EKF. There is a significant lower estimation accuracy for both filters at 0C than at ambient temperature, caused by the lower OCV slope and higher internal resistance at low temperatures, leading to each voltage measurement containing less information.

There are three ways to further extend the study. Firstly, the adoption of the Adaptive Extended Kalman Filter (AEKF) in order to enhance its robustness over variations in the operating conditions; secondly, extending the ECM to a 3RC model structure so that to more efficiently model the dynamics of slow diffusion in low temperature and low SoC region, thirdly, using a swarm intelligence based method (PSO) in optimization of the ECM parameters as either a replacement or as an alternative method to the Levenberg-Marquardt algorithm in order to obtain the global optimal.

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