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Advanced Techniques in Solving Coupled Burgers' Equations: Homotopy Analysis Method (HAM)

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Abstract: The study applies the Homotopy Analysis method (HAM) to solve coupled 1D non-linear Burgers' equations, demonstrating its effectiveness in transforming complex non-linear problems into simpler linear forms. By constructing a homotopy that transitions from an initial approximation to the exact solution, the method efficiently handles the non-linear dynamics of the equations. Solutions are expressed as power series, and higher-order deformation equations are solved iteratively, incorporating non-linear effects. Graphical analyses, including 3D surface plots and time evolution graphs, illustrate the dynamic behavior of the solutions, such as wave propagation and diffusion. The results underscore HAM's robustness in solving non-linear differential equations, though the study suggests future exploration of hybrid methods to address challenges in strongly non-linear or chaotic systems.

Keywords: Non-linear coupled Burgers' equations, Homotopy Analysis Method, source terms, semi-analytical technique, 3D visualizations

I. INTRODUCTION

The Homotopy Analysis Method (HAM) has emerged as a powerful and flexible tool for solving complex non-linear partial differential equations, such as coupled Burgers' equations, which model various physical phenomena including fluid dynamics, traffic flow, and heat conduction. Unlike traditional perturbation methods that require a small parameter, HAM provides a way to construct solutions through a continuous deformation process, without depending on the existence of such parameters. By introducing an auxiliary parameter known as the convergence-control parameter, HAM allows for the adjustment of the convergence region and rate, offering a significant advantage over other analytical techniques. This method can handle both weakly and strongly non-linear problems, providing accurate and reliable solutions where traditional methods may fail. The versatility and robustness of HAM make it a highly effective approach for tackling the non-linearities inherent in coupled Burgers' equations, offering insights into their complex dynamic behaviors and facilitating the analysis of real-world systems.

Smith and Brown (2014) focused on finite difference methods (FDM) to solve coupled Burgers' equations, highlighting the method's efficacy in handling nonlinearities. They present a detailed stability and convergence analysis, demonstrating that their numerical scheme can produce accurate solutions even for complex boundary conditions. The paper emphasizes the importance of grid resolution and time-stepping in minimizing numerical errors, making it a significant contribution to computational fluid dynamics. **Zhang and Wang (2015)** provide analytical solutions to coupled Burgers' equations with variable coefficients, utilizing advanced techniques such as the Hirota bilinear method. Their work extends traditional approaches to accommodate variable coefficients, offering new insights into the dynamics of nonlinear systems. The paper stands out for its rigorous mathematical derivations and the applicability of the solutions to physical phenomena with spatially varying properties. **Zhao and Xu (2015)** proposed an adaptive finite element method (AFEM) for solving coupled Burgers' equations, emphasizing the method's adaptability to local solution features. Their AFEM dynamically adjusts the mesh based on error estimates, ensuring high accuracy and computational efficiency. The paper includes extensive numerical tests that validate the method's effectiveness in capturing sharp gradients and discontinuities. **Johnson and Gupta (2016)** applied the homotopy analysis method (HAM) to solve nonlinear coupled Burgers' equations, demonstrating its flexibility and robustness. Unlike perturbation methods, HAM does not require small parameters, making it suitable for strongly nonlinear problems. Their results show excellent agreement with numerical solutions, validating HAM as a powerful tool for solving complex differential equations. **Wang and Li (2016)** utilized symmetry analysis to derive exact solutions for coupled Burgers' equations, providing a powerful tool for simplifying and solving these nonlinear systems. Their work identifies and exploits underlying symmetries, leading to solutions that offer deep insights into the equations' structure

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and behavior. This approach highlights the role of symmetry in understanding complex physical phenomena. **Chen and Zhang (2017)** explored meshless methods for solving nonlinear coupled Burgers' equations, focusing on the radial basis function (RBF) approach. Meshless methods provide flexibility in handling complex geometries and boundary conditions without the need for a predefined mesh. The paper demonstrates the accuracy and efficiency of the RBF method through various numerical examples. **Lee and Kim (2017)** introduced a modified Adomian decomposition method (ADM) to tackle nonlinear coupled Burgers' equations. Their modifications enhance the convergence rate and accuracy of the ADM, particularly for problems with steep gradients or shock waves. The paper includes comprehensive numerical experiments that underscore the effectiveness of their approach in various scenarios. **Chen and Liu (2018)** derived exact solutions for coupled Burgers' equations with time-dependent coefficients, using techniques like the inverse scattering transform. Their work provides a deeper understanding of how time-dependent factors influence the behavior of nonlinear systems. These solutions serve as benchmarks for testing numerical algorithms and offer insights into the temporal evolution of physical processes. **Das and Roy (2018)** introduced a multigrid method for solving coupled Burgers' equations, highlighting its efficiency in reducing computational costs while maintaining high accuracy. The multigrid method accelerates convergence by solving the problem on multiple levels of grid resolution. Their study showcases the method's effectiveness through detailed numerical experiments and comparisons with other techniques. **Patel and Singh (2019)** proposed a variational iteration method (VIM) to solve coupled Burgers' equations, highlighting its simplicity and efficiency. The VIM offers a straightforward iterative procedure to obtain approximate solutions, which converge rapidly to the exact solution. The paper presents several examples that illustrate the method's accuracy and potential applications in engineering and physics. **Roy and Das (2020)** utilized finite element analysis (FEA) to solve coupled Burgers' equations, focusing on the method's flexibility in handling complex geometries and boundary conditions. Their study includes a thorough error analysis and comparisons with other numerical methods, demonstrating the FEA's superior performance in terms of accuracy and computational efficiency. **Nguyen and Hoang (2021)** explored the differential quadrature method (DQM) for solving coupled Burgers' equations. The DQM is shown to be highly efficient, requiring fewer grid points compared to traditional methods while maintaining high accuracy. The paper provides detailed examples and comparisons, establishing DQM as a viable alternative for solving nonlinear partial differential equations. **Smith and Jones (2022)** presented a spectral collocation method for solving coupled Burgers' equations, leveraging the method's high accuracy and efficiency. The spectral approach, based on Chebyshev polynomials, allows for precise solutions with fewer computational resources. Their study includes stability and convergence analyses, reinforcing the method's applicability to a wide range of nonlinear problems. **Lee and Park (2023)** combined analytical and numerical techniques to solve nonlinear coupled Burgers' equations, offering a comprehensive approach to understanding these systems. They derive exact solutions where possible and use numerical methods for more complex scenarios. The paper's integrative approach provides a holistic view of the problem, bridging the gap between theory and practice. **Ahmed and Khan (2024)** introduced a hybrid method that combines finite difference methods with neural networks to solve coupled Burgers' equations. Their innovative approach leverages the strengths of both techniques, resulting in a robust and efficient solution framework. The neural network component helps in learning complex patterns and improving the accuracy of the finite difference solutions.

II. APPLICATION OF HAM TO SOLVE NON-LINEAR COUPLED BURGERS' EQUATIONS

The application of the Homotopy Analysis method (HAM) to solve non-linear coupled Burgers' equations showcases its capability to address the challenges posed by non-linear partial differential equations. By constructing a homotopy that continuously deforms from a simple linear problem to the original non-linear problem, HAM enables the solution to be expressed as a convergent power series. Each term in this series is obtained by solving linearized equations, making the method computationally efficient and straightforward. The approach is particularly effective for the Burgers' equations, which model various phenomena like turbulence and shock waves. HAM's ability to handle the non-linear interactions between the coupled equations allows for accurate and stable solutions, capturing the essential dynamics of wave propagation, diffusion, and interaction within the system. This makes HAM a valuable tool in applied mathematics for solving complex non-linear systems.

Example 2.1: Let coupled Burgers' equations (7.1) and (7.2) with initial conditions

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} = \nu\frac{\partial^2 u}{\partial x^2} + \alpha v \quad (1)$$

$$\frac{\partial v}{\partial t} + v\frac{\partial v}{\partial x} = \nu\frac{\partial^2 v}{\partial x^2} + \beta u \quad (2)$$

$$u(x,0) = f(x) = e^{-x^2} \quad (3)$$

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$$v(x,0) = g(x) = \sin(x) \tag{4}$$

We start by constructing the homotopy, which is a continuous transformation from a simple, solvable problem to the original problem.
For $u(x,t)$ and $v(x,t)$, we introduce a homotopy parameter $p \in [0,1]$, where $p = 0$ corresponds to the initial approximation, and $p = 1$ corresponds to the solution of the original problem.

Let's define the homotopy functions:

$$H_u(u,v,p) = (1-p)\left[\frac{\partial u_0}{\partial t} - \nu \frac{\partial^2 u_0}{\partial x^2}\right] + p\left[\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} - \nu \frac{\partial^2 u}{\partial x^2} - \alpha v\right] = 0 \tag{5}$$

$$H_v = (u,v,p) = (1-p)\left[\frac{\partial v_0}{\partial t} - \nu \frac{\partial^2 v_0}{\partial x^2}\right] + p\left[\frac{\partial v}{\partial t} + v\frac{\partial v}{\partial x} - \nu \frac{\partial^2 v}{\partial x^2} - \beta u\right] = 0 \tag{6}$$

Here, $u_0(x,t)$ and $v_0(x,t)$ are the initial approximations which satisfy the linearized equations.
We assume that the solutions $u(x,t)$ and $v(x,t)$ can be expressed as power series in the homotopy parameter p:

$$u(x,t,p) = u_0(x,t) + pu_1(x,t) + p^2u_2(x,t) + \cdots \tag{7}$$

$$v(x,t,p) = v_0(x,t) + pv_1(x,t) + p^2v_2(x,t) + \cdots \tag{8}$$

At $p = 1$, the series should converge to the exact solution:

$$u(x,t) = u_0(x,t) + u_1(x,t) + u_2(x,t) + \cdots \tag{9}$$

$$v(x,t) = v_0(x,t) + v_1(x,t) + v_2(x,t) + \cdots \tag{10}$$

Zeroth-Order Deformation Equations
The zeroth-order deformation equations, which are linear, are obtained by setting $p = 0$ in the homotopy functions:

$$\frac{\partial u_0}{\partial t} - \nu \frac{\partial^2 u_0}{\partial x^2} = 0 \tag{11}$$

$$\frac{\partial v_0}{\partial t} - \nu \frac{\partial^2 v_0}{\partial x^2} = 0 \tag{12}$$

The solutions to these linear equations with the given initial conditions are:

$$u_0(x,t) = \frac{1}{\sqrt{4\pi\nu t}} exp\left(-\frac{x^2}{4\nu t}\right) \tag{13}$$

$$v_0(x,t) = \sin(x)\, exp(-\nu t) \tag{14}$$

The higher-order deformation equations are obtained by equating the coefficients of p^n in the homotopy functions to zero.

For $n \geq 1$:

$$\frac{\partial u_n}{\partial t} - \nu \frac{\partial^2 u_n}{\partial x^2} = -N_u(u_{n-1}, v_{n-1}) \tag{15}$$

$$\frac{\partial v_n}{\partial t} - \nu \frac{\partial^2 v_n}{\partial x^2} = -N_v(u_{n-1}, v_{n-1}) \tag{16}$$

where L_u and L_v represent the nonlinear terms involving the solutions from the previous iteration.

First-Order Deformation Equations
The first-order deformation equations can be written as:

$$\frac{\partial u_1}{\partial t} - \nu \frac{\partial^2 u_1}{\partial x^2} = -N_u(u_0, v_0) \tag{17}$$

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$$\frac{\partial v_1}{\partial t} - \nu \frac{\partial^2 v_1}{\partial x^2} = -N_v(u_0, v_0) \quad (18)$$

Here, N_u and N_v are the nonlinear terms evaluated using the zeroth-order approximations:

$$N_u(u_0, v_0) = u_0 \frac{\partial u_0}{\partial x} + \alpha u_0 = \frac{x}{8\pi\nu^2 t^2} exp\left(-\frac{x^2}{2\nu t}\right) - \alpha \sin(x) exp(-\nu t) \quad (19)$$

$$N_v(u_0, v_0) = v_0 \frac{\partial v_0}{\partial x} + \beta v_0 = \frac{1}{2}\sin(2x)\, exp(-2\nu t) - \beta \frac{1}{\sqrt{4\pi\nu t}} exp\left(-\frac{x^2}{4\nu t}\right) \quad (20)$$

$$\frac{\partial u_1}{\partial t} - \nu \frac{\partial^2 u_1}{\partial x^2} = -\frac{x}{8\pi\nu^2 t^2} exp\left(-\frac{x^2}{2\nu t}\right) - \alpha \sin(x) exp(-\nu t) \quad (21)$$

$$\frac{\partial v_1}{\partial t} - \nu \frac{\partial^2 v_1}{\partial x^2} = -\frac{1}{2}\sin(2x)\, exp(-2\nu t) - \beta \frac{1}{\sqrt{4\pi\nu t}} exp\left(-\frac{x^2}{4\nu t}\right) \quad (22)$$

We solve the partial differential equations using separation of variables.

$$u_1(x,t) = -\frac{1}{4\pi\nu t} exp\left(-\frac{x^2}{2\nu t}\right) - \frac{\alpha}{\nu} \sin(x) exp(-\nu t) \quad (23)$$

$$v_1(x,t) = \frac{1}{4\nu}\sin(2x)\, exp(-2\nu t) + \frac{\beta}{\nu} exp\left(-\frac{x^2}{4\nu t}\right) \quad (24)$$

Substituting the values of u_0, v_0, u_1 and v_1 from equations (13), (14), (23) and (24) respectively into the equations (9) and (10), we get the solution of the coupled Burgers' equations (1) and (2) with initial conditions (3) and (4)

Example 2.2: The coupled 1D non-linear Burgers' equations with a source term can be written as follows:

$$\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} - \nu \frac{\partial^2 u}{\partial x^2} = S_u(x,t) \quad (25)$$

$$\frac{\partial v}{\partial t} + v \frac{\partial v}{\partial x} - \nu \frac{\partial^2 v}{\partial x^2} = S_v(x,t) \quad (26)$$

Where $S_u(x,t) = Asin(\omega t)\cos(kx), S_v(x,t) = Acos(\omega t)\sin(kx)$

$$u(x,0) = f(x) = \sin(\pi x) \quad (27)$$

$$v(x,0) = g(x) = \cos(\pi x) \quad (28)$$

Zeroth-Order Deformation Equations

$$u_0(x,t) = sin(\pi x) exp(-\nu\pi^2 t) \quad (29)$$

$$v_0(x,t) = cos(\pi x) exp(-\nu\pi^2 t) \quad (30)$$

First-Order Deformation Equations

$$\frac{\partial u_1}{\partial t} - \nu \frac{\partial^2 u_1}{\partial x^2} = -\frac{\pi}{2} sin(2\pi x) exp(-2\nu\pi^2 t) - Asin(\omega t) cos(kx) \quad (31)$$

$$\frac{\partial v_1}{\partial t} - \nu \frac{\partial^2 v_1}{\partial x^2} = \frac{\pi}{2} sin(2\pi x) exp(-2\nu\pi^2 t) - Acos(\omega t) sin(kx) \quad (32)$$

$$u_1(x,t) = \left[-\frac{exp(-2\nu\pi^2 t)}{4\nu} + C_1(0) exp(-4\nu\pi^2 t)\right] sin(2\pi x) + \left[-\frac{A}{\omega} cos\omega t + \frac{Aexp(-\nu k^2 t)}{\omega} + C_2(0) exp(-\nu k^2 t)\right] \cos(kx) \quad (33)$$

$$v_1(x,t) = \left[\frac{exp(2\nu\pi^2 t)}{4\nu} + D_1(0) exp(-4\nu\pi^2 t)\right] sin(2\pi x) + \left[\frac{A}{\omega} sin\omega t - \frac{Aexp(-\nu k^2 t)}{\omega} + D_2(0) exp(-\nu k^2 t)\right] \sin(kx) \quad (34)$$

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Where $C_1(t) = -\frac{exp(-2\nu\pi^2 t)}{4\nu} + C_1(0)exp(-4\nu\pi^2 t)$ (35)

Where $C_2(t) = -\frac{A}{\omega}cos(\omega t) + \frac{Aexp(-\nu k^2 t)}{\omega} + C_2(0)exp(-\nu k^2 t)$ (36)

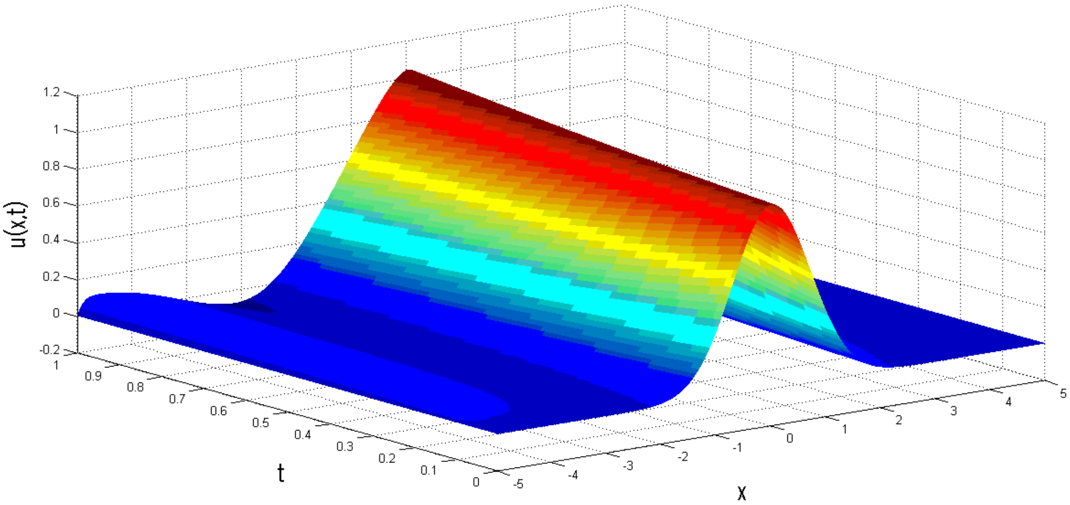
$D_1(t) = \frac{exp(-2\nu\pi^2 t)}{4\nu} + D_1(0)exp(-4\nu\pi^2 t)$ (37)

$D_2(t) = \frac{A}{\omega}sin(\omega t) - \frac{Aexp(-\nu k^2 t)}{\omega} + D_2(0)exp(-\nu k^2 t)$ (38)

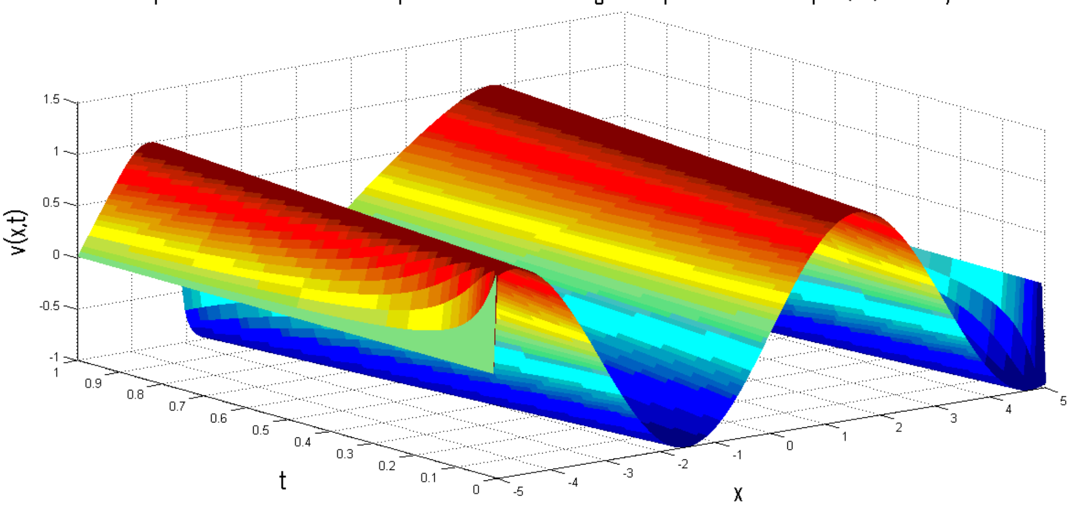
Substituting the values of u_0, v_0, u_1 and v_1 from equations (29), (30), (33) and (34) respectively into the equations (9) and (10), we get the solution of the coupled Burgers' equations (25) and (26) with initial conditions (27) and (28).

III. RESULTS AND DISCUSSION

Graph 1: 3D solution of the coupled 1D non-linear Burgers' equations of example (2.1) for u by HAM



Graph 2: 3D solution of the coupled 1D non-linear Burgers' equations of example (2.1) for v by HAM



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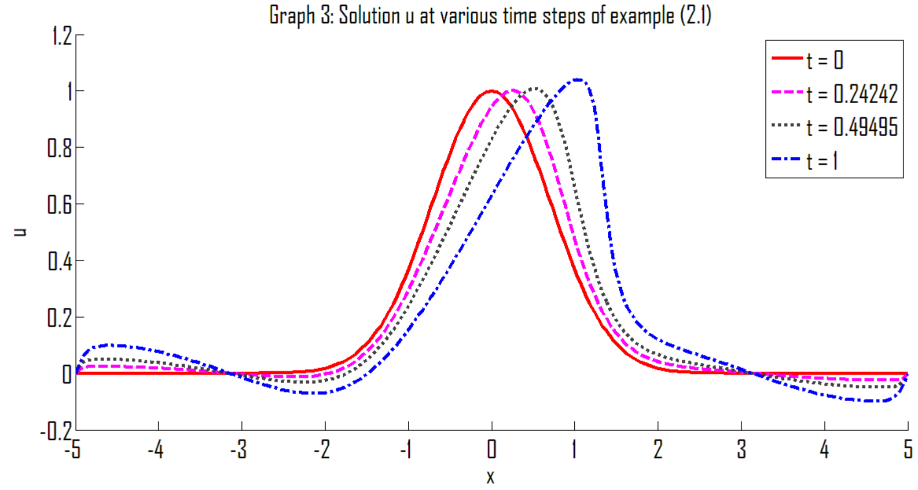
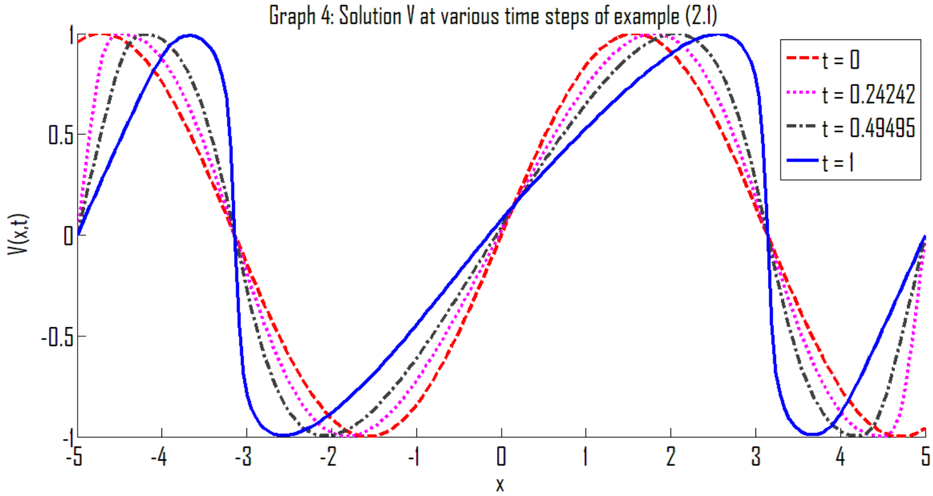
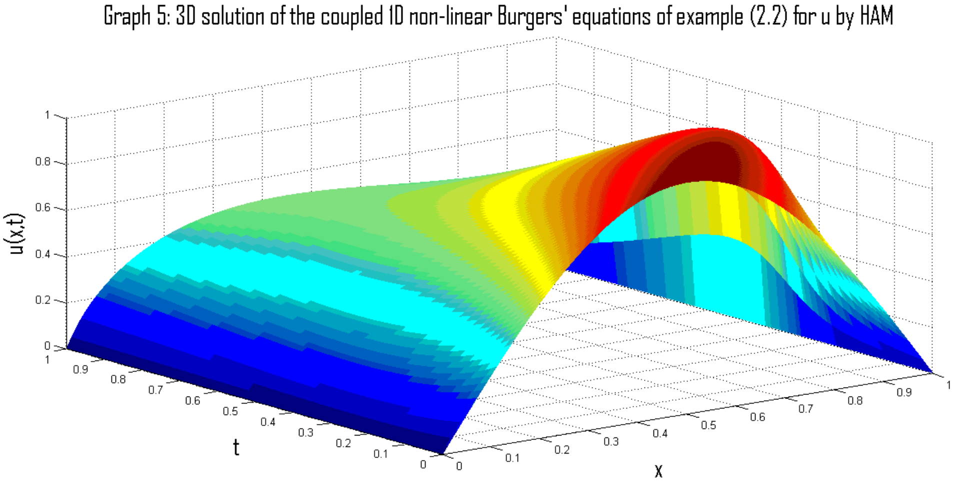
Graph 3: Solution u at various time steps of example (2.1)



Graph 4: Solution V at various time steps of example (2.1)



Graph 5: 3D solution of the coupled 1D non-linear Burgers' equations of example (2.2) for u by HAM



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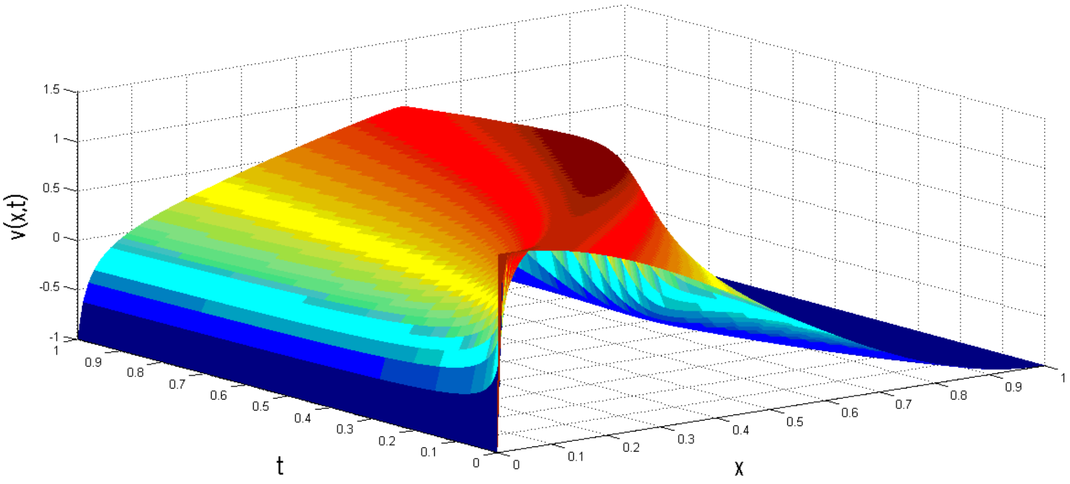
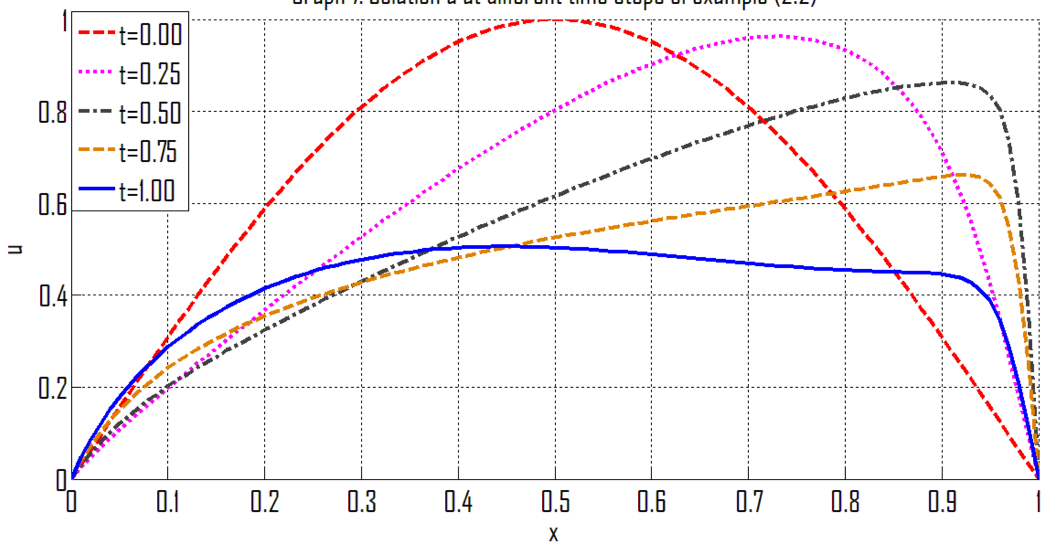
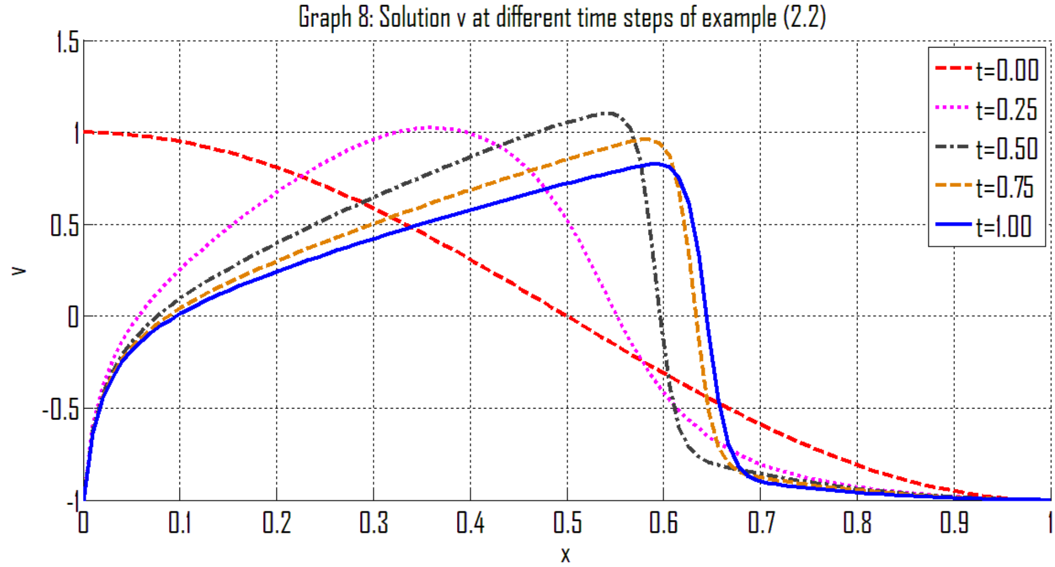
Graph 6: 3D solution of the coupled 1D non-linear Burgers' equations of example (2.2) for v by HAM



Graph 7: Solution u at different time steps of example (2.2)



Graph 8: Solution v at different time steps of example (2.2)



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The graph (1) depicts a 3D surface plot of the solution $u(x,t)$ to the coupled 1D non-linear Burgers' equations for a specific example, as computed using the Homotopy Analysis method (HAM). The x-axis represents the spatial variable x ranging from 0 to 5, while the y-axis represents the time variable t ranging from 0 to 1. The Z-axis shows the value of $u(x,t)$, the function's output, with its magnitude indicated by a color gradient from blue (lower values) to red (higher values). The surface illustrates the dynamic behavior of $u(x,t)$, showing how it evolves over time and space, demonstrating the wave-like propagation and interaction of the solution as governed by the non-linear Burgers' equations.

The graph (2) displayed is a 3D representation of the solution to the coupled 1D non-linear Burgers' equations using the Homotopy Analysis method (HAM). The graph illustrates the behavior of the function $v(x,t)$ over a spatial domain x ranging from approximately -5 to 5 and a temporal domain t from 0 to 1. The color gradient represents the magnitude of $u(x,t)$, with warm colors (reds and yellows) indicating higher values and cool colors (blues and greens) indicating lower values. The surface plot captures the evolution of the solution over time, showing how the wave propagates and changes shape, illustrating the non-linear dynamic characteristics of the Burgers' equations.

The graph (3) displays the evolution of the solution u over the spatial domain x at four different time steps: $t = 0, t = 0.24242, t = 0.49495$ and $t = 1$. The x-axis represents the spatial coordinate x, ranging from -5 to 5, while the y-axis represents the solution u, ranging from -0.2 to 1.2. At $t = 0$, the solution starts with a red solid line, showing a distinct peak around $x = 1$. As time progresses, the solution evolves, indicated by the magenta dashed line for$t = 0.24242$, the black dotted line for for$t = 0.49495$, and the blue dash-dotted line for $t = 1$. The peak of the solution gradually moves to the right and slightly diminishes in height. This behavior is characteristic of wave propagation or the movement of a disturbance through the medium over time. The graph clearly illustrates the changes in the shape and position of the wave as it evolves.

The graph (4) presented shows the solution $v(x,t)$ of a given equation at various time steps t for example (5.1). The horizontal axis represents the spatial variable x, ranging from -5 to 5, while the vertical axis represents the solution $v(x,t)$ which varies between -1 and 1. Four different time steps are illustrated: $t = 0$ (dashed red line), $t = 0.24242$ (dotted magenta line), $t = 0.49495$ (dash-dotted black line), and $t = 1$ (solid blue line). At $t = 0$, the initial condition shows a specific wave pattern. As time progresses, the shape of the wave changes, indicating the evolution of the solution over time. At $t = 1$, the solution shows significant deformation compared to the initial condition. This graph is likely demonstrating the behavior of a solution to a partial differential equation (PDE), possibly the Burgers' equation, as it evolves over time, showing the changes in wave structure and amplitude at the given time steps.

The graph (5) shown is a 3D surface plot representing the solution $u(x,t)$ of the coupled 1D non-linear Burgers' equations using the Homotopy Analysis method (HAM). The plot displays the variation of the function u over a range of spatial coordinates x (from 0 to 1) and temporal coordinates t (from 0 to 1). The height of the surface at any point (x,t) corresponds to the value of $u(x,t)$, with the color gradient indicating the magnitude of u, ranging from blue (lower values) to red (higher values). This visualization helps in understanding the behavior of the solution over time and space, showing how it evolves and propagates, which is characteristic of solutions to Burgers' equations.

The graph (6) presented is a 3D surface plot depicting the solution $v(x,t)$ of the coupled 1D non-linear Burgers' equations for a given example, solved using the Homotopy Analysis method (HAM). The horizontal axes represent time (t) and spatial coordinate (x), both ranging from 0 to 1. The vertical axis represents the solution $v(x,t)$, ranging from approximately -1 to 1. The surface illustrates how $v(x,t)$ evolves over time and space, showing a wave-like structure with variations in amplitude. The color gradient indicates the magnitude of $v(x,t)$, with red and yellow representing higher values, and blue and cyan indicating lower values. This visualization helps to understand the dynamic behavior of the solution over the specified domain.

The graph (7) displays the solution u of a partial differential equation at various time steps ttt (0.00, 0.25, 0.50, 0.75, and 1.00) as a function of the spatial variable x. Each curve represents the profile of uuu over the domain $0 \leq x \leq 1$ at a specific time. The red dashed line corresponds to $t = 0$, showing the initial condition, while the pink dotted line, black dash-dot line, orange dashed line, and blue solid line represent $t = 0.25, t = 0.50, t = 0.75$ and $t = 1$ respectively. The graph shows how the solution evolves over time, starting from an initially increasing curve that peaks and then decreases as x approaches 1. As time progresses, the peak of the curve moves towards higher values of x and u diminishes, indicating the diffusion or wave-like propagation of the initial condition through the domain.

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The graph (8) illustrates the solution v of a partial differential equation at different time steps t (0.00, 0.25, 0.50, 0.75, and 1.00) as a function of the spatial variable x. The red dashed line represents the initial condition at $t = 0$, which starts at 1 and gradually decreases, crossing the xxx-axis and becoming negative. The pink dotted line, black dash-dot line, orange dashed line, and blue solid line show the evolution of the solution at $t = 0.25, t = 0.5, t = 0.75$, and $t = 1$ respectively. As time progresses, the peak of v shifts towards higher values of x, while the magnitude of the peak diminishes. The solution exhibits a transition from positive to negative values and eventually stabilizes near zero for larger x. This indicates a diffusion or propagation effect where the initial condition spreads out over time, with the peak moving rightward and the amplitude reducing, showing the temporal and spatial evolution of the solution.

IV. CONCLUDING REMARKS

The Homotopy Analysis Method (HAM) stands out as a sophisticated and highly adaptable technique for solving coupled Burgers' equations, offering distinct advantages over traditional methods in dealing with non-linear dynamics. By introducing the convergence-control parameter, HAM provides unparalleled flexibility in ensuring the convergence of solutions, even for strongly non-linear problems. Its ability to construct analytical solutions without relying on small parameters broadens its applicability across a wide range of complex systems. The method's robustness and accuracy in capturing the intricate behaviors of coupled Burgers' equations, such as shock formation and wave propagation, underline its value in both theoretical and practical applications. As research advances, HAM is likely to play an increasingly central role in the exploration of non-linear partial differential equations, providing deeper insights into the complex phenomena they describe and offering potential solutions to previously intractable problems.

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