

Confidence Level based on Logarithmic Einstein Aggregation Approach for Fuzzy Matrix Games with Multi-Expert Evaluation

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Abstract: In real world situations decision-making usually depends on the opinions of multiple experts with different level of knowledge and confidence. Namarta et al. [18] introduced a method for solving intuitionistic fuzzy matrix games using Intuitionistic Fuzzy Einstein Interactive Weighted Averaging (IFEIWA) Operator to aggregate the weightage of multiple experts for payoffs. However, a significant limitation of this method is that they assume all the experts have 100% confidence in their payoffs. But in real world competition scenarios experts have varying degrees of familiarity with specific strategies. For example; they might be very experienced yet still feel unsure about particular strategy. To address this limitation, this paper proposes a significant method for solving intuitionistic fuzzy game problems by introducing CLIFEWA (Confidence Logarithmic Intuitionistic Fuzzy Einstein Weighted Averaging) Operator. To show the superiority, validity and practical applicability of proposed method illustrative example has been given.

Keywords: Intuitionistic Fuzzy Sets, Fuzzy Matrix Game, Aggregation Operator

I. INTRODUCTION

Across diverse fields of study, game theory provides the essential mathematical structure required to analyse strategic interactions and competitive decision-making. The core objective in these competitive scenarios is the identification of optimal strategies that minimize potential losses while maximizing gains for opponents with conflicting interests. But classical game theory assumes that the payoffs associated with different strategies are known precisely. However, in many real-world situations, such an assumption is unrealistic due to vagueness, incomplete information and human judgment. To address these uncertainties, researchers have increasingly integrated fuzzy set theory into strategic modelling. Zadeh [24] initially proposed fuzzy sets to handle satisfaction and dissatisfaction, which was later expanded by Atanassov [1-3] into Intuitionistic Fuzzy Sets (IFSs).

Bellman and Zadeh [4] first introduced the concept of decision-making in a fuzzy environment by modeling goals and constraints as fuzzy sets. Later, Li [14, 15] proposed multi-objective programming approaches and solution methodologies for matrix games with fuzzy payoffs, establishing a strong connection between fuzzy optimization and game theory. Further developments were made by Cevikel and Ahlatcioglu [7], who formulated models for solving matrix games involving fuzzy goals and payoffs. Chakeri and Sheikholeslam [8] extended the concept of Nash equilibrium to fuzzy environments, providing a more flexible interpretation of equilibrium under uncertainty. In addition, Li and Cheng [16] introduced computational procedures based on auxiliary fuzzy linear programming models, while Li and Nan [17] proposed nonlinear programming approaches for matrix games with intuitionistic fuzzy payoffs. Nan and Li [19] further explored matrix games with intuitionistic fuzzy goals, presenting linear programming techniques to obtain optimal strategies. Vijay et al. [21, 22] contributed by developing fuzzy relational approaches and demonstrating the equivalence between fuzzy matrix games and primal-dual fuzzy linear programming problems. Moreover Brikaa et al. [5, 6] introduced advanced models incorporating fuzzy rough sets and indeterminacy-resolving functions to handle multi-criteria and complex game scenarios.

Another important direction in the literature is the use of aggregation operators in decision-making. Xu [23] introduced intuitionistic fuzzy aggregation operators, which enable the combination of multiple fuzzy values into a single representative value. Garg [9-13] further extended this area by proposing various interactive aggregation operators and improved score functions for intuitionistic fuzzy environments, enhancing their applicability in multi-criteria decision-making problems. Rahman et al. [20] presented the aggregation operators based on Einstein t-norm and t-conorm such as CLIFEWG Operator, CLIFEOWG Operator, CLIFEHG Operator, CLIFEWA Operator, CLIFEHA Operator. Namarta et al. [18] has developed effective methodology for solving intuitionistic fuzzy matrix games. Specifically, Namarta et al. [18] introduced a bi-parametric approach that aggregates the judgments of multiple experts by assigning weights to their opinions.

But in practical decision-making scenarios, the reliability of information or expert opinion plays a crucial role. Different experts may provide assessments with varying degrees of confidence depending on their experience, knowledge and available data. Ignoring this confidence level may lead to less robust or biased decisions. Therefore, it is essential to incorporate a mechanism that not only model uncertainty through intuitionistic fuzzy sets but also reflects the confidence associated with these evaluations. Motivated by this need, this paper focuses on the development of a fuzzy intuitionistic matrix game framework that integrates confidence levels into the payoff structure. So, in this paper, a significant method is proposed for solving intuitionistic fuzzy game problems by using CLIFEWA Operator.

The remainder of this paper is structured as follows: Section 2 reviews preliminaries; Section 3 describes the extension of the Intuitionistic Fuzzy Matrix Game with CLIFEWA Operator; Section 4 represents the proposed method to solve fuzzy intuitionistic matrix game based on the weightage and confidence Level of different experts and Section 5 demonstrates the superiority of this proposed method through a numerical example. Finally, a concrete conclusion has been given in Section 6.

II. PRELIMINARIES

In this section, basic definitions and Einstein operational laws of intuitionistic fuzzy sets are discussed. Also, CLIFEWA operator is defined in this section.

2.1 Intuitionistic Fuzzy Sets

Definition 2.11 [1]: Following the foundational work of Atanassov [2], an Intuitionistic Fuzzy Set (IFS) \tilde{A} defined over a fixed universe X is represented by $\tilde{A} = \{(x, \mu_{\tilde{A}}(x), \nu_{\tilde{A}}(x)); x \in X\}$ In this expression, the functions $\mu_{\tilde{A}}(x)$ and $\nu_{\tilde{A}}(x)$ denote the membership and non-membership degrees of an element $x \in X$ respectively. Also $0 \leq \mu_{\tilde{A}}(x) + \nu_{\tilde{A}}(x) \leq 1 \forall x \in X$. Furthermore, the uncertainty or 'hesitation degree' associated with an element x is calculated as $\pi_{\tilde{A}}(x) = 1 - \mu_{\tilde{A}}(x) - \nu_{\tilde{A}}(x)$.

Definition 2.12 [1]: An intuitionistic fuzzy numbers (IFN) \tilde{A} has the following properties:

- \tilde{A} is convex for membership function $\mu_{\tilde{A}}(x)$, i.e. $\mu_{\tilde{A}}(\lambda x_1 + (1 - \lambda)x_2) \geq \min[\mu_{\tilde{A}}(x_1), \mu_{\tilde{A}}(x_2)]$ for $x_1, x_2 \in R; \lambda \in [0,1]$
- \tilde{A} is concave for non-membership function $\nu_{\tilde{A}}(x)$, i.e. $\nu_{\tilde{A}}(\lambda x_1 + (1 - \lambda)x_2) \leq \max[\nu_{\tilde{A}}(x_1), \nu_{\tilde{A}}(x_2)]$ for $x_1, x_2 \in R; \lambda \in [0,1]$
- \tilde{A} is normal.

2.2 Einstein Operational Laws for IFNs

Based on the Einstein t-norm and t-conorm, the basic operational laws for two Intuitionistic Fuzzy Numbers (IFNs) $\tilde{x}_1 = \langle \mu_1, \nu_1 \rangle$ and $\tilde{x}_2 = \langle \mu_2, \nu_2 \rangle$, as utilized by Rahman et al. [20] are defined as follows:

- **Einstein Sum (\oplus):**

$$x_1 \oplus x_2 = \left(\frac{\mu_1 + \mu_2}{1 + \mu_1 \mu_2}, \frac{\nu_1 \nu_2}{1 + (1 - \nu_1)(1 - \nu_2)} \right)$$

- **Einstein Product (\otimes):**

$$x_1 \otimes x_2 = \left(\frac{\mu_1 \mu_2}{1 + (1 - \mu_1)(1 - \mu_2)}, \frac{\nu_1 + \nu_2}{1 + \nu_1 \nu_2} \right)$$

2.3 CLIFEWA (Confidence Logarithmic Intuitionistic Fuzzy Einstein Weighted Averaging) Operator

The approach established by Rahman et al. [20] i.e., the CLIFEWA operator is utilised to aggregate a set of intuitionistic fuzzy numbers (IFNs). This operator integrates the confidence levels of decision-makers into the logarithmic Einstein weighted averaging framework, defined as follows:

Let $\tilde{x}_i = \langle \mu_i, \nu_i \rangle$ be a set of n intuitionistic fuzzy numbers (IFNs) representing expert opinions where $\varphi_i \in [0, 1]$ is

the confidence level of the i th expert and $w = (w_1, w_2, w_3, \dots, w_n)^T$ is their weight vector such that $\sum_{i=1}^n w_i = 1$ then

CLIFEWA Operator can be defined as

$$\left(\frac{\prod_{i=1}^n (2 - \log_{\alpha} \mu_i)^{\varphi_i \omega_i} - \prod_{i=1}^n (\log_{\alpha} \mu_i)^{\varphi_i \omega_i}}{\prod_{i=1}^n (2 - \log_{\alpha} \mu_i)^{\varphi_i \omega_i} + \prod_{i=1}^n (\log_{\alpha} \mu_i)^{\varphi_i \omega_i}} \right) \cdot \left(\frac{2 \prod_{i=1}^n (\log_{\alpha} (1 - \nu_i))^{\varphi_i \omega_i}}{\prod_{i=1}^n (2 - \log_{\alpha} (1 - \nu_i))^{\varphi_i \omega_i} + \prod_{i=1}^n (\log_{\alpha} (1 - \nu_i))^{\varphi_i \omega_i}} \right)$$

Where $\alpha \neq 1$ and $0 \leq \alpha \leq \min \{ \mu_i, (1 - \nu_i) \} \leq 1$

III. EXTENSION OF THE INTUITIONISTIC FUZZY MATRIX GAME WITH CLIFEWA OPERATOR

In this section extension of the existing method [18] is described. As we know in real world decision-making information is always uncertain. When experts evaluate payoffs in a game, they often face problems such as lack of complete data and differences in their level of expertise. For instance, a player may understand the game strategies well but may not be fully sure about the opponents in next move. In traditional fuzzy methods, this feeling of uncertainty is lost. As in the same way although the method proposed by Namarta et al. [18] is mathematically sound for general group decisions. It operates under the assumption that every expert is 100% confident in the values they provide. But in a competitive matrix game, this assumption can lead to risky strategies if high weighted expert provides an opinion based on guess. Therefore, it is mathematically necessary to incorporate confidence level. This helps to distinguish between an expert's overall knowledge and their confidence in particular decision or strategy

To address this limitation this paper extends Namarta et al. [18] method by introducing CLIFEWA Operator. By doing so it is ensured that final strategy is not just based on who the experts are but also on how sure they are about each specific payoff. So, such type of problems in which confidence level of experts are sought cannot be solved by Namarta et al. [18] is shown in an example 5.

IV. PROPOSED METHOD TO SOLVE FUZZY INTUITIONISTIC MATRIX GAME BASED ON THE WEIGHTAGE AND CONFIDENCE LEVEL OF DIFFERENT EXPERTS

In this competitive world a player never depends upon the opinion of one expert but also considers the opinion of more than one expert by taking expert's overall knowledge and their confidence in a particular strategy. In this situation an aggregation operator is an important tool used to aggregate the opinion of different experts. The method given by Namarta et al. [18] is only applicable where every expert is 100% confident in the payoff values which they provide but in reality, this assumption can lead to risk strategies if highly weighted expert provides an opinion based on guess only. So, to keep in mind a new method is proposed to solve fuzzy matrix game by introducing confidence level of experts also. In this method CLIFEWA operator is used to aggregate the payoffs of intuitionistic fuzzy matrix game. The steps are as under:

Step 1: Let A_1 and A_2 be two players. Let $P = \{ \alpha_1, \alpha_2, \dots, \alpha_m \}$ and $Q = \{ \beta_1, \beta_2, \dots, \beta_n \}$ be the set of pure strategies of player A_1 and A_2 respectively. Let $\{ E_1, E_2, \dots, E_n \}$ be the set of experts who gave their judgements regarding the strategies of each player. Thus, the payoffs of players A_1 given by different experts with $m \times n$ strategies can be expressed in the intuitionistic fuzzy game matrices as follows:

$$P^{E_1} = \begin{pmatrix} \langle \mu_{11}^{E_1}, \nu_{11}^{E_1} \rangle & \dots & \langle \mu_{1n}^{E_1}, \nu_{1n}^{E_1} \rangle \\ \vdots & \ddots & \vdots \\ \langle \mu_{m1}^{E_1}, \nu_{m1}^{E_1} \rangle & \dots & \langle \mu_{mn}^{E_1}, \nu_{mn}^{E_1} \rangle \end{pmatrix}$$

$$P^{E_2} = \begin{pmatrix} \langle \mu_{11}^{E_2}, \nu_{11}^{E_2} \rangle & \dots & \langle \mu_{1n}^{E_2}, \nu_{1n}^{E_2} \rangle \\ \vdots & \ddots & \vdots \\ \langle \mu_{m1}^{E_2}, \nu_{m1}^{E_2} \rangle & \dots & \langle \mu_{mn}^{E_2}, \nu_{mn}^{E_2} \rangle \end{pmatrix}$$

$$\vdots$$

$$P^{E_n} = \begin{pmatrix} \langle \mu_{11}^{E_n}, \nu_{11}^{E_n} \rangle & \dots & \langle \mu_{1n}^{E_n}, \nu_{1n}^{E_n} \rangle \\ \vdots & \ddots & \vdots \\ \langle \mu_{m1}^{E_n}, \nu_{m1}^{E_n} \rangle & \dots & \langle \mu_{mn}^{E_n}, \nu_{mn}^{E_n} \rangle \end{pmatrix}$$

Step 2: Now aggregate the above matrices in which payoffs are given by different experts with the use of CLIFEWA Operator [20] defined in subsection 2.3

After aggregating the intuitionistic fuzzy payoffs of the matrices, the resultant matrix is

$$P^{EAgg} = \begin{bmatrix} \langle \mu_{11}^{EAgg}, \nu_{11}^{EAgg} \rangle & \dots & \langle \mu_{1n}^{EAgg}, \nu_{1n}^{EAgg} \rangle \\ \vdots & \ddots & \vdots \\ \langle \mu_{m1}^{EAgg}, \nu_{m1}^{EAgg} \rangle & \dots & \langle \mu_{mn}^{EAgg}, \nu_{mn}^{EAgg} \rangle \end{bmatrix}$$

Step 3: The bi-objective mathematical programming model of player A₁ by using the resultant matrix is as follow:

$$Z = \max\{\psi\}, \min\{\xi\}$$

$$s.t \begin{cases} 1 - \prod_{i=1}^m (1 - \mu_{ij}^{EAgg})^{p_i} \geq \psi \quad (j = 1, 2, 3, \dots, n), \\ \prod_{i=1}^m (\nu_{ij}^{EAgg})^{p_i} \leq \xi \quad (j = 1, 2, 3, \dots, n), \\ 0 \leq \psi + \xi \leq 1, \psi \geq 0, \xi \geq 0, \\ \sum_{i=1}^m p_i = 1, \\ p_i \geq 0, \quad (i = 1, 2, 3, \dots, m) \end{cases}$$

$$\text{where } \max\{\psi\} = \max \left\{ \min_{1 \leq j \leq n} \left(1 - \prod_{i=1}^m (1 - \mu_{ij}^{EAgg})^{p_i} \right) \right\}$$

$$\text{and } \min\{\xi\} = \min \left\{ \max_{1 \leq j \leq n} \left(\prod_{i=1}^m (\nu_{ij}^{EAgg})^{p_i} \right) \right\}$$

Now the above aggregation of bi-objective mathematical programming model of player A₁ can be converted into linear programming model is as follows:

$$Z = \min \{w\}$$

$$s.t \begin{cases} \sum_{i=1}^m [\lambda \ln(1 - \mu_{ij}^{EAgg}) + (1 - \lambda) \ln(\nu_{ij}^{EAgg})] p_i \leq w \quad ; (j = 1, 2, \dots, n) \\ \sum_{i=1}^m p_i = 1, \quad p_i \geq 0 \quad (i = 1, 2, \dots, m) \\ w \text{ is an unrestricted} \\ 0 \leq 1 - \psi \leq 1, \quad 0 \leq \xi \leq 1, \quad \lambda \in [0, 1] \end{cases}$$

Similarly the linear programming model of player A₂ which is the dual of the above linear programming of player A₁ is constructed as under:

Let $q_1, q_2, q_3, \dots, q_n$ and u be dual variables corresponding to above constraints

$$Z^* = \max \{0q_1 + 0q_2 + 0q_3 + \dots + 0q_n + u\}$$

$$s.t \begin{cases} \sum_{j=1}^n [\lambda \ln(1 - \mu_{ij}^{EAgg}) + (1 - \lambda) \ln(\nu_{ij}^{EAgg})] q_j \geq u \quad ; (i = 1, 2, \dots, m) \\ \sum_{j=1}^n q_j = 1, \\ q_j \geq 0 \quad (j = 1, 2, \dots, n) \\ \text{and } u \text{ is an unrestricted} \end{cases}$$

Step 4: Now apply MATLAB 7.0 to solve the above linear programming problems of both the players. Then the expected value of game and optimal strategies of both the players are obtained.

V. NUMERICAL EXAMPLE

To address the limitations of existing models that assume absolute expert certainty, the above method has proposed. This approach integrates confidence levels into the aggregation process using the CLIFEWA operator to ensure a more realistic representation of competitive payoffs. Such type of some problems in which opinions and confidence level of more than one expert are sought, cannot be solved by Namarta et al. [18] as shown below in an example.

Let company A_1 and A_2 be two leading smartphone manufactures competing to dominate the premium 5G market segment. To attract customers and to increase their market shares both companies follows 3 identical strategies. Strategy I involves the implementation of aggressive pricing models including significant discounts and trade in-centives to capture cost-sensitive customers. Strategy II focus on technological differentiation by investing in hardware innovation such as foldable displays and AI integrated camera sensors. Strategy III adopts an ecosystem centring approach bundling primary devices with premium accessories (e.g. smart watches or earbuds) to enhance the overall customer value proposition. Since market is unstable and customers behaviour is unpredictable, so companies have consulted two independent market analyst says expert E_1 and E_2 to evaluate potential outcomes. The experts have depicted their opinion using intuitionistic fuzzy sets. According to their experience each expert E_1 and E_2 give weightage of 0.7 and 0.3 respectively and also according to their knowledge they have given confidence level 0.5 and 0.9 respectively. The payoff matrices are obtained for company A_1 according to the opinion of two experts E_1 and E_2 are as follows:

$$P^{E_1} = \begin{matrix} & I & II & III \\ I & \langle 0.7, 0.2 \rangle & \langle 0.6, 0.3 \rangle & \langle 0.9, 0.05 \rangle \\ II & \langle 0.6, 0.15 \rangle & \langle 0.5, 0.4 \rangle & \langle 0.4, 0.4 \rangle \\ III & \langle 0.85, 0.05 \rangle & \langle 0.8, 0.1 \rangle & \langle 0.7, 0.3 \rangle \end{matrix}$$

$$P^{E_2} = \begin{matrix} & I & II & III \\ I & \langle 0.85, 0.1 \rangle & \langle 0.7, 0.2 \rangle & \langle 0.8, 0.1 \rangle \\ II & \langle 0.5, 0.1 \rangle & \langle 0.6, 0.2 \rangle & \langle 0.5, 0.2 \rangle \\ III & \langle 0.7, 0.1 \rangle & \langle 0.9, 0.05 \rangle & \langle 0.6, 0.2 \rangle \end{matrix}$$

Step 2: Now apply CLIFEWA Operator to aggregate all the payoffs of above matrices. It is given that the weights of experts E_1 and E_2 are 0.7 and 0.3 respectively. Also consider the confidence level of experts are 0.5 and 0.9 respectively. We get the intuitionistic fuzzy values \tilde{x}_{ij} 's which represents the aggregated values of the payoff matrix at the pure strategies $\alpha_i \in P$ and $\beta_j \in Q$ are as follows:

$$\tilde{x}_{11} = \langle 0.641, 0.278 \rangle$$

Where

$$\tilde{x}_{11} = \left\langle \frac{((2 - \log_{0.2} 0.7)^{0.5 \times 0.7})((2 - \log_{0.2} 0.85)^{0.5 \times 0.7}) - ((\log_{0.2} 0.7)^{0.5 \times 0.7})(\log_{0.2} 0.85)^{0.5 \times 0.7}}{((2 - \log_{0.2} 0.7)^{0.5 \times 0.7})((2 - \log_{0.2} 0.85)^{0.5 \times 0.7}) + ((\log_{0.2} 0.7)^{0.5 \times 0.7})(\log_{0.2} 0.85)^{0.5 \times 0.7}}, \frac{2((\log_{0.2}(1 - 0.2))^{0.9 \times 0.3})(\log_{0.2}(1 - 0.1))^{0.9 \times 0.3}}{((2 - \log_{0.2}(1 - 0.2))^{0.9 \times 0.3})((2 - \log_{0.2}(1 - 0.1))^{0.9 \times 0.3}) + ((\log_{0.2}(1 - 0.2))^{0.9 \times 0.3})(\log_{0.2}(1 - 0.1))^{0.9 \times 0.3}} \right\rangle$$

In the similar way we can find the other payoff values. Then the aggregated payoff matrix is as under:

$$P^{E_{agg}} = \begin{matrix} & I & II & III \\ I & \langle 0.641, 0.278 \rangle & \langle 0.582, 0.337 \rangle & \langle 0.678, 0.197 \rangle \\ II & \langle 0.4356, 0.2517 \rangle & \langle 0.4236, 0.4344 \rangle & \langle 0.3237, 0.4334 \rangle \\ III & \langle 0.6610, 0.1894 \rangle & \langle 0.7224, 0.1825 \rangle & \langle 0.5440, 0.3644 \rangle \end{matrix}$$

Step 3: Write the linear programming problem of above aggregated matrix for both the players which are dual of each other. The Linear programming problem of player A_1 and Player A_2 are constructed as under:

For Player A_1

$$Z = \min \{w\}$$

Subject to

$$\lambda \ln(1 - 0.641) + (1 - \lambda) \ln(0.278)] p_1 + \lambda \ln(1 - 0.4356) + (1 - \lambda) \ln(0.2517)] p_2 + \lambda \ln(1 - 0.6610) + (1 - \lambda) \ln(0.1894)] p_3 \leq w ,$$

$$\lambda \ln(1 - 0.582) + (1 - \lambda) \ln(0.337)] p_1 + \lambda \ln(1 - 0.4236) + (1 - \lambda) \ln(0.4344)] p_2 + \lambda \ln(1 - 0.7224) + (1 - \lambda) \ln(0.1825)] p_3 \leq w ,$$

$$\lambda \ln(1 - 0.678) + (1 - \lambda) \ln(0.197)] p_1 + \lambda \ln(1 - 0.3237) + (1 - \lambda) \ln(0.4334)] p_2 + \lambda \ln(1 - 0.5440) + (1 - \lambda) \ln(0.3644)] p_3 \leq w ,$$

$$p_1 + p_2 + p_3 = 1$$

$p_1, p_2, p_3 \geq 0$ and w is an unrestricted

The above problem can be written as follows:

$$Z = \min \{w\}$$

Subject to

$$\lambda \ln(0.359) + (1 - \lambda) \ln(0.278)] p_1 + \lambda \ln(0.5644) + (1 - \lambda) \ln(0.2517)] p_2 + \lambda \ln(0.339) + (1 - \lambda) \ln(0.1894)] p_3 \leq w ,$$

$$\lambda \ln(0.418) + (1 - \lambda) \ln(0.337)] p_1 + \lambda \ln(0.5764) + (1 - \lambda) \ln(0.4344)] p_2 + \lambda \ln(0.2776) + (1 - \lambda) \ln(0.1825)] p_3 \leq w , \tag{1}$$

$$\lambda \ln(0.322) + (1 - \lambda) \ln(0.197)] p_1 + \lambda \ln(0.6763) + (1 - \lambda) \ln(0.4334)] p_2 + \lambda \ln(0.456) + (1 - \lambda) \ln(0.3644)] p_3 \leq w ,$$

$$p_1 + p_2 + p_3 = 1$$

$p_1, p_2, p_3 \geq 0$ and w is an unrestricted

For Player A₂

$$Z^* = \max \{u\}$$

Subject to

$$\lambda \ln(0.359) + (1 - \lambda) \ln(0.278)] q_1 + \lambda \ln(0.418) + (1 - \lambda) \ln(0.337)] q_2 + \lambda \ln(0.332) + (1 - \lambda) \ln(0.197)] q_3 \geq u ,$$

$$\lambda \ln(0.5644) + (1 - \lambda) \ln(0.2517)] q_1 + \lambda \ln(0.5764) + (1 - \lambda) \ln(0.4344)] q_2 + \lambda \ln(0.6763) + (1 - \lambda) \ln(0.4334)] q_3 \geq u , \tag{2}$$

$$\lambda \ln(0.339) + (1 - \lambda) \ln(0.1894)] q_1 + \lambda \ln(0.2776) + (1 - \lambda) \ln(0.1825)] q_2 + \lambda \ln(0.456) + (1 - \lambda) \ln(0.3644)] q_3 \geq u ,$$

$$q_1 + q_2 + q_3 = 1$$

$q_1, q_2, q_3 \geq 0$ and u is an unrestricted

Step 4: For different values of $\lambda \in [0,1]$, apply MATLAB 7.0 to solve linear programming problem of (1) and (2) for both the players. The optimal solution is shown below in the table 1

Table 1: Optimal solution and expected payoffs of the aggregated fuzzy matrix game of both players

λ	p^{*T}	w^*	q^{*T}	u^*	$E(p^*, q^*) = p^* P^{EAgg} q^{*T}$
0.3	(0.5823, 0, 0.4177)	(-1.2537)	(0,0.4921,0.5079)	(-1.2537)	(0.5836, 0.2751)
0.7	(0.3139, 0.6861, 0)	(-1.0267)	(0.4312,0,0.5688)	(-1.0267)	(0.5806, 0.2895)
0.5	(0.3514, 0.6486, 0)	(-1.1061)	(0.4389,0,0.5611)	(-1.1061)	(0.5785, 0.2894)

Thus we obtain the maximin strategies p^* of player A₁ and minimax strategies q^* of player A₂ as well as expected payoffs of the fuzzy matrix game P^{EAgg} . For example, if $\lambda = 0.5$, the maximin strategies p^* of player A₁ is (0.3514, 0.6486, 0) and minimax strategies q^* of player A₂ is (0.4389, 0, 0.5611) and the corresponding expected payoffs are (0.5785, 0.2894). So the result obtained at $\lambda = 0.5$ reveals that Company A₁ will use his Ist strategy of providing heavy discounts with probability of 0.3514 and also uses his IInd strategy to provide technological differentiation by investing in hardware innovation with probability of 0.6486. On the other hand Company A₂ will use his Ist strategy to provide an offer of significant discounts with probability of 0.4389 and also uses IIIRD strategy to adopt ecosystem centric approach with probability of 0.5611.

VI. CONCLUSION

This paper addresses a critical gap in the existing literature regarding multi-expert decision-making within the framework of fuzzy matrix games. While the method proposed by Namarta et al. [18], established a robust foundation for aggregating group judgments using the IFEIWA operator, they operated under the unrealistic assumption that every expert is 100% certain about their judgments. By introducing the CLIFEWA operator, this paper evolves the model to systematically

account for varying degrees of expert confidence. It concluded that neglecting an expert's internal hesitation can lead to biased strategic outcomes; however, by integrating confidence as a dynamic parameter. This method makes the final decision much more accurate for real-life business and competition. A numerical example has been given to show the validity of the proposed method. Furthermore, this proposed method will be practically applicable in highly uncertain competitive environments.

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