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Binary Contra Regular ^ Generalized Continuous Functions in Binary Topological Spaces

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Abstract: In this article, we introduce and investigate the notion of binary contra regular ^ generalized continuous (shortly μ_b contra r^g-continuous), binary almost contra regular ^ generalized continuous (shortly μ_b almost contra r^g-continuous) functions and discussed their relationships with other binary contra continuous functions and obtained some of their characteristics.

Keywords: Binary contra r^g continuous, binary almost contra r^g continuous, $T^\wedge_{1/2}$ space

MSC: 54A05, 54C05, 54A99

I. INTRODUCTION

Based on a study, the concept of binary topology from X to Y is introduced by the authors [4]. Further the concepts of binary closure, binary interior and binary continuity also introduced by them.

If A is a subset of X and B is a subset of Y, then the topological structures on X and Y provide a little information about the ordered pair (A, B). In 2011, S. Jothi S.N. [4] introduced a single structure which carries the subsets of X as well as the subsets of Y for studying the information about the ordered pair (A, B) of subsets of X and Y. Such a structure is called a binary structure from X to Y. Mathematically a binary structure from X to Y is defined as a set of ordered pairs (A, B) where $A \subseteq X$ and $B \subseteq Y$. Already binary regular ^ generalized closed sets and binary regular ^ generalized continuous functions are introduced by [8] in general topological spaces.

In continuation, in the present paper we have defined and explored several properties of binary contra regular ^ generalized and almost contra regular ^ generalized continuous functions. Also some of its properties have been discussed.

II. PRELIMINARIES

Definition 2.3[2]: Let X and Y be any two non empty sets. A binary generalized topology from X to Y is a binary structure $\mu_b \subseteq P(X) \times P(Y)$ that satisfies the following axioms:

(i) $(\phi,\phi) \in \mu_b$ and $(X,Y) \in \mu_b$.

(ii) $(A_1 \cap A_2, B_1 \cap B_2) \in \mu_b$ whenever (A_1,B_1) and $(A_2,B_2) \in \mu_b$

(iii) If $\{(A_\alpha,B_\alpha) : \alpha \in \Delta\}$ is a family of members of μ_b, then $(\cup A_\alpha, \cup B_\alpha) \in \mu_b$.

If μ_b is a binary generalized topology from X to Y then the triplet (X, Y, μ_b) is called a binary generalized topological space and the members of μ_b are called binary generalized open sets.

The compliment of an element of $P(X) \times P(Y)$ is defined component wise. That is the binary compliment of (A, B) is (X – A, Y – B). The elements of X×Y are called the binary points of the binary topological space (X, Y, μ_b). If X = Y then μ_b is called a binary topology on X in which case we write (X, μ_b) as a binary space.

Definition 2.2[4]: Let (X, Y, μ_b) be a binary generalized topological space and $A \subseteq X$, $B \subseteq Y$. Then (A, B) is called binary generalized closed if (X – A, Y – B) is binary generalized open.

Definition 2.3[4]: Let (A,B), (C,D) $\in P(X) \times P(Y)$. Then

(i) $(A, B) \subseteq (C, D)$ if $A \subseteq C$ and $B \subseteq D$.

(ii) $(A, B) \cup (C, D) =(A \cup C, B \cup D)$.

(iii) $(A, B) \cap (C, D) =(A \cap C, B \cap D)$.

Definition 2.4[4]: Let (X, Y, μ_b) be a binary generalized topological space and $(x, y) \in X \times Y$, then a subset (A, B) of (X, Y) is called a binary generalized neighbourhood of (x, y) if there exists a binary generalized open set (U, V) such that $(x, y) \in (U, V) \subseteq (A, B)$.

Definition 2.5[4]: Let (X, Y, μ_b) be a binary generalized topological space and A $\subseteq$ X, B $\subseteq$ Y. Let $(A, B)^{1*} = \cap\{A_\alpha :(A_\alpha, B_\alpha)$is binary generalized closed and $(A, B) \subseteq (A_\alpha, B_\alpha)\}$ and $(A, B)^{2*} = \cap\{B_\alpha :(A_\alpha, B_\alpha)$is binary generalized closed and $(A, B) \subseteq (A_\alpha, B_\alpha)\}$. Then the pair $((A, B)^{1*},(A, B)^{2*})$ is called the binary generalized closure of (A,B) and denoted by μ_bCl(A,B).

Remark 2.6[2]: The binary generalized closure μ_bCl(A,B) is binary generalized closed such that (A,B)$\subseteq\mu_b$Cl(A,B).

Definition 2.7[2]: Let (X, Y, μ_b) be a binary generalized topological space and A $\subseteq$ X,B $\subseteq$ Y. Let $(A, B)^{1°} = \cup\{A_\alpha : (A_\alpha, B_\alpha)$ is binary generalized open and $(A_\alpha, B_\alpha) \subseteq (A, B)\}$ and $(A, B)^{2°} = \cup\{B_\alpha : (A_\alpha, B_\alpha)$is binary generalized open and $(A_\alpha, B_\alpha) \subseteq (A, B)\}$. Then the pair $((A, B)^{1°},(A, B)^{2°})$ is called the binary generalized interior of (A,B) and denoted by μ_bInt(A,B).

Remark 2.8[2]: The binary generalized interior μ_bInt(A,B) is binary generalized open such that μ_bInt(A,B) $\subseteq$ (A,B).

Definition 2.9[2]: A subset (A,B) of topological space (X, Y, μ_b) is called a binary regular open set **(shortly μ_b regular open set)** if (μ_bInt($\mu_b$cl A,B)) $\subseteq$ (A,B).

Definition 2.10[7]:Let (X,Y,μ_b) be a binary topological space. Let (A,B) $\subseteq$ (X,Y). Then (A,B) is called a binary regular ^ generalized closed set **(shortly μ_b r^g-closed set)** if there exists a binary regular open set (U,V) such that $\mu_b$gcl(A,B) $\subseteq$ (U,V) whenever (A,B) $\subseteq$ (U,V).

Definition 2.11[8]: Let (Z,η) be a topological space and (X,Y, μ_b) be a binary topological space. Then the map f: Z → X×Y is called a **binary regular^generalized continuous (shortly r^g-continuous) function** if f^{-1}(A,B) is r^g closed in (Z,η) for every binary closed set (A,B) in (X,Y, μ_b).

Let us introduce some definitions to binary topology which already exists in general topology.

Definition 2.12[7]: Let (Z,η) be a topological space and (X,Y, μ_b) be a binary topological space. Then the map f:Z → X×Y is called

(i) a **binary g-continuous** function if f^{-1}(A,B) is gclosed in (Z,η) for every binary closed set (A,B) in (X, Y, μ_b).

(ii) a **binary g*-continuous** function if f^{-1}(A,B) is g*closed in (Z,η) for every binary closed set (A,B) in (X, Y, μ_b).

(iii) a **binary rwg-continuous** function if f^{-1}(A,B) is rwg closed in (Z,η) for every binary closed set (A,B) in (X, Y, μ_b).

(iv) a **binary rgw-continuous** function if f^{-1}(A,B) is rgw closed in (Z,η) for every binary closed set (A,B) in (X, Y, μ_b).

Definition 2.13[3]: Let (Z,η) be a topological space and (X,Y, μ_b) be a binary topological space. Then the map f:Z → X×Y is called a binary RC continuous map if f^{-1}(A,B) is regular closed in (Z,η) for each binary open set (A,B) in (X, Y, μ_b).

Definition 2.14[3]: A function f:Z → X×Y is called a **binary regular set connected** if f^{-1}(A,B) is clopen in (Z,η) for each binary regular open set (A,B) in (X, Y, μ_b).

Definition 2.15[6]: A topological space (Z,η) is said to be

(i) a **T^$_{1/2}$ space[6]** if every r^g closed set is gclosed.

(ii) **locally indiscrete[3]** if every open subset of Z is closed.

III. BINARY CONTRA REGULAR ^ GENERALIZED CONTINUOUS FUNCTIONS

Definition 3.1: A function f: Z →X×Y is said to be a binary contra continuous **(shortly μ_b contra continuous)** function if the inverse image of every binary open set (U,V) of X×Y is closed set in (Z,η).

Definition 3.2: A function f: Z →X×Y is said to be a binary contra regular^generalized continuous **(shortly μ_b contra r^g continuous)** function if the inverse image of every binary open set (U,V) of X×Y is r^g closed set in (Z,η).

Example 3.3: Let Z = {a,b,c,d}, X = $\{x_1,x_2\}$, Y = $\{y_1,y_2\}$, η = {ϕ, Z,{a},{c},{a,c},{c,d}. {a,c,d}}, μ_b = $\{(\phi,\phi),(\{x_1\},\{y_2\}),(\{x_2\},\{y_1\}), (X,Y)\}$. Define f: Z→X×Y as f(a)= $(\{x_1\},\{y_2\})$ = f(d), f(b) = $(\{x_2\},\{y_1\})$ = f(c). Then $f^{-1}\{(\phi,\phi)\} = \phi, f^{-1}(\{x_1\},\{y_2\}) = \{a,d\}, f^{-1}(\{x_2\},\{y_1\}) = \{b,c\}, f^{-1}(X,Y) = Z$. Here f is μ_b contra r^g continuous function.

Definition 3.4[3]: A binary space (X,Y,μ_b) is **binary locally indiscrete** if every binary open subset of (X,Y,μ_b) is binary closed.

Definition 3.5: A binary topological space (X,Y,μ_b) is **binary r^g locally indiscrete** if every binary r^g open subset of (X,Y,μ_b) is binary closed.

Theorem 3.6: Let f: (Z,η) → (X,Y,μ_b) be a function.

(i) If f is binary r^g continuous and (Z,η) is r^g locally indiscrete then f is binary contra r^g continuous.

(ii) If f is binary r^g continuous and Z is T^1/2 space then f is binary contra r^g continuous.

Proof: (i) Suppose f is μ_b r^g continuous. Let Z be locally indiscrete and let V be a μ_b open set in (X,Y,μ_b). Since f is μ_b r^g continuous, $f^{-1}(V)$ is r^g open in (Z,η). By hypothesis, $f^{-1}(V)$ is closed in Z. Every closed set is r^g closed hence f is μ_b contra r^g continuous.

(ii) Let f be μ_b r^g continuous and Z is T^1/2 space. Let V be a μ_b open set in (X,Y,μ_b). Since f is μ_b r^g continuous, $f^{-1}(V)$ is r^g closed in (Z,η). Since Z is T^1/2 space, $f^{-1}(V)$ is gclosed in (Z,η). Every gclosed set is r^g closed hence f is binary contra r^g continuous.

Theorem 3.7: Every binary RC continuous function is binary contra r^g continuous function.
Proof: Straight forward.
Remark 3.8: The converse of the above theorem need not be true as shown in the following example.
Example 3.9: Let Z = {a,b,c,d}, X = {x_1,x_2}, Y = {y_1,y_2}, η = {ϕ, Z,{a},{c},{a,c},{c,d}, {a,c,d}}, μ_b = {(ϕ,ϕ),(ϕ,{y_2}),(X,Y)}. Clearly η is a topology on Z and μ_b is a topology from X to Y. Define f: Z→X×Y as f(a)= (ϕ,ϕ), f(b) = (ϕ,{y_2}), f(c) = ({x_2},ϕ). Clearly f is a binary contra r^g continuous function but it is not a binary RC continuous function since f^{-1}((ϕ,{y_2}) = {b} is r^g closed but it is not regular closed set in (Z,η).

Theorem 3.10: Every binary contra continuous function is binary contra r^g continuous function.
Proof: Straight forward from the definition.
Remark 3.11: The converse of the above theorem need not be true as shown in the following example.
Example 3.12: Let Z = {a,b,c}, X = {x_1,x_2}, Y = {y_1,y_2}, η = {ϕ, Z,{a},{b},{a,b}}, μ_b = {(ϕ,ϕ),({x_2},{y_2}),(X,Y)}. Clearly η is a topology on Z and μ_b is a topology from X to Y. Define f: Z→X×Y as f(a)= (ϕ,{y_2}), f(b) = ({x_2},ϕ) and f(c) = (ϕ,ϕ). Then f is binary contra r^g continuous but it is not binary contra continuous function since f^{-1}({x_2},{y_2}) = {a,b} is r^g closed but it is not closed in (Z,η).

Theorem 3.13: Every binary contra g continuous, binary contra g* continuous function is binary contra r^g continuous function.
Proof: Obvious from the definition.
Remark 3.14: The converse of the above theorem need not be true as shown in the following example.
Example 3.15: Let Z = {a,b,c}, X = {x_1,x_2}, Y = {y_1,y_2}, η = {ϕ, Z,{b},{a,b}}, μ_b ={(ϕ,ϕ),({x_2},{y_1}), (X,Y)}. Clearly η is a topology on Z and μ_b is a topology from X to Y. Define f: Z→X×Y as f(a)= ({x_2},{y_1}), f(b) = f(c) = (ϕ,ϕ). Then f is a binary contra r^g continuous function but it is not a binary contra g continuous and g* continuous function since the inverse image of ({x_2},{y_1}) = {a} is r^g closed but is not g closed and g*closed in (Z,η).

Theorem 3.16: Every binary contra r^g continuous function is rwg continuous, binary contra rgw continuous function..
Proof: Obvious from the definition.
Remark 3.17: The converse of the above theorem need not be true as shown in the following example.
Example 3.18: Let Z = {a,b,c,d}, X = {x_1,x_2}, Y = {y_1,y_2}, η = {ϕ, Z,{a},{c},{a,c},{c,d}, {a,c,d}}, μ_b ={(ϕ,ϕ),({x_2},{y_2}),(X,Y)}. Clearly η is a topology on Z and μ_b is a topology from X to Y. Define f: Z→X×Y as f(a)= (ϕ,ϕ),({x_2},{y_2}) = f(b) f(c) = (X,Y). Then f is binary contra r^g continuous function but it is not both binary contra rwg and rgw continuous functions since f^{-1}({x_2},{y_2}) = {b} is r^g closed and it is not both rwg closed and rgw closed in (Z,η).

Theorem 3.19: Suppose R^GO(Z,η) is closed under arbitrary unions. If f: Z → X×Y is binary contra r^g continuous function and X×Y is regular, then f is binary r^g continuous.
Proof: Let x be an arbitrary point of (Z,η) and (A,B) be a binary open set of X×Y containing f(x). The regularity of Z implies that there exists an open set (U,V) containing f(x) in X×Y such that $\mu_b$gcl(U,V) ⊆ (A,B). Since f is binary contra r^g continuous then there exists Q∈R^GO(Z,η) such that f(Q) ⊆ $\mu_b$gcl(U,V) ⊆ (A,B). Thus f is binary r^g continuous function.

IV. BINARY ALMOST CONTRA REGULAR ^ GENERALIZED CONTINUOUS FUNCTIONS

Definition 4.1: A function f: Z →X×Y is said to be a binary almost contra continuous **(shortly μ_b almost contra continuous)** function if the inverse image of every binary regular open set (U,V) of X×Y is closed set in (Z,η).

Definition 4.2: A function f: Z →X×Y is said to be a binary almost contra regular^generalized continuous **(shortly μ_b almost contra r^g continuous)** function if the inverse image of every binary regular open set (U,V) of X×Y is r^g closed set in (Z,η).

Theorem 4.3: Every binary almost contra continuous function is binary almost contra r^g continuous function.
Proof: Straight forward from the definition.
Remark 4.4: The converse of the above theorem need not be true as shown in the following example.

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Example 4.5: Let Z = {a,b,c,d}, X = {x_1,x_2}, Y = {y_1,y_2}, η = {ϕ, Z,{a},{c},{a,c},{c,d}, {a,c,d}}, μ_b ={(ϕ,ϕ), ({x_1},ϕ), ({x_1},{y_2}), ({x_2},{y_1}),(X,{y_1}), (X,Y)}. Clearly η is a topology on Z and μ_b is a topology from X to Y. Define f: Z→X×Y as f(a)= (ϕ,{y_1}), ({x_1},{y_2}) = f(b) f(c) = ({x_2},ϕ) f(d) = ({x_1},ϕ).Then f is binary almost contra r^g continuous function but it is not binary almost contra continuous function.

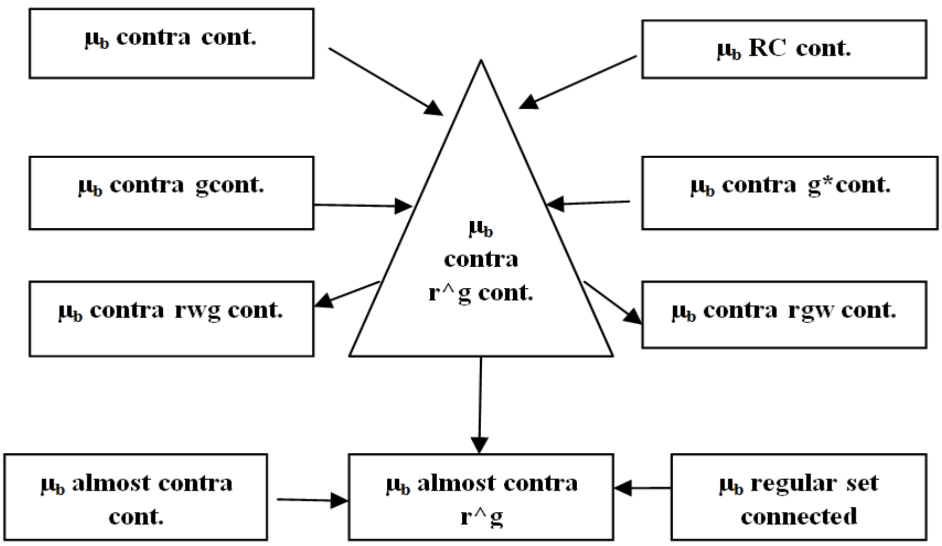
Theorem 4.6: Every binary contra r^g continuous function is binary almost contra r^g continuous function.
Proof: Obvious from the fact that every binary regular open set is binary open.
Remark 4.7: The converse of the above theorem need not be true as seen in the following example.
Example 4.8: Let Z = {a,b,c} η = {Z,ϕ,{b},{a,b}} X= {a,b}, Y = {1,2} μ_b = {(ϕ,ϕ), (ϕ,{1}), ({a},{1}),({b},{1}),(X,{1}),(X,Y)}. Clearly μ_b is a binary topology from X to Y. Define f: Z → X×Y as f(a) = f(c) = (ϕ,ϕ) f(b) = ({a},{1}). Then f is binary almost contra r^g continuous function but it is not a binary contra r^g continuous function.

Theorem 4.9: Every binary regular set connected function is binary almost contra r^g continuous function.
Proof: Straight forward.
Remark 4.10: The converse of the above theorem need not be true as seen in the following example.
Example 4.11: Let Z = {a,b,c}, η= {Z,(ϕ,{a}, ({b},{a,b})}, X = {x_1,x_2}, Y = {y_1,y_2}, μ_b = {(ϕ,ϕ), ({x_1},{y_1}), ({x_2},{y_2})(X,Y)}. Clearly η is a topology on Z and μ_b is a topology from X to Y. Define f: Z→X×Y as f(a)= ({x_1},ϕ),({x_1},Y) = f(b) f(c) = ({x_2},{y_2}). Then f is binary almost contra r^g continuous function but it is not binary regular set connected since f^{-1}({x_2},{y_2} = {c}is r^g closed in (Z,η) but it is not clopen.

Theorem 4.12: Suppose r^g closed sets of Z is closed under arbitrary unions. The following statements are equivalent for a given function f: Z → X×Y.
(i) f is binary almost contra r^g continuous function.
(ii) For every binary regular closed subset (A,B) of X×Y,f^{-1}(A,B)∈R^GO(Z,η).
(iii) For each x∈Z and each binary closed set (A,B) in X×Y containing f(x), there exists an r^g open set U in Z containing x such that f(U) ⊆ (A,B).

Proof: (i) ⟹(ii): Let (A,B) be a binary regular closed set. Then (X-A,Y-B) is a binary regular open set. Since f is almost contra continuous function, the inverse image of (X-A,Y-B) ∈ R^GC(Z,η). Hence f^{-1}(A,B) ∈ R^GO(Z,η).
(ii) ⟹(i) and (iii) ⟹ (i) are obvious.
(ii) ⟹ (iii): Let (A,B) be a binary regular closed set in X×Y containing f(x). f^{-1}(A,B) ∈ R^GO(Z,η) and x ∈ f^{-1}(A,B). Taking U = f^{-1}(A,B), f(U) ⊂ (A,B).
(iii) ⟹ (ii): Let (A,B) ∈ RC(X×Y) and x ∈f^{-1}(A,B). From (iii), there exists an r^g open set U in Z containing x such that U ⊂ f^{-1}(A,B). We have f^{-1}(A,B) = ∪ {U: x∈f^{-1}(A,B)}. Thus f^{-1}(A,B) is r^g open.

The above discussions are implicated in the following diagram.



In the diagram, A → B represents A implies B but not conversely.

V. CONCLUSION

In this paper, using binary r^g closed sets, we have defined binary r^g continuous functions and analyzed some of its properties. Furthermore binary r^g continuous functions has been compared with some of other binary continuous functions. This concept can be extended in future.

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