UGC Approved Journal

IARJSET

ISSN (Online) 2393-8021
ISSN (Print) 2394-1588

International Advanced Research Journal in Science, Engineering and Technology

ISO 3297:2007 Certified

Vol. 4, Issue 7, July 2017

A General Fixed Point Theorem in Multiplicative Metric Spaces

Avinash Chandra Upadhyaya

Department of Mathematics, Deenbandhu Chhotu Ram University of Science and Technology, Murthal, Sonepat, Haryana, India

Abstract: In this paper, we prove a general fixed point theorem that generalizes various results present in multiplicative fixed point literature.

Keywords: Multiplicative metric spaces, rational inequality, fixed point.

Mathematics Subject Classification: 47H10, 54H25.

1. INTRODUCTION AND PRELIMINARIES

It is well know that the set of positive real numbers $\mathbb{R}_+$ is not complete according to the usual metric. To overcome this problem, in 2008, Bashirov et al. [2] introduced the concept of multiplicative metric spaces as follows:

Definition1.1. ([2]) Let X be a non-empty set. A multiplicative metric is a mapping d: X×X → $\mathbb{R}^+$ satisfying the following conditions:

(i) $d(x, y) \geq 1$ for all $x, y \in X$ and $d(x, y) = 1$ if and only if x=y;
(ii) $d(x, y) = d(y, x)$ for all $x, y \in X$;
(iii) $d(x, y) \leq d(x, z).\ d(z, y)$ for all $x, y, z \in X$ (multiplicative triangle inequality).

Then mapping d together with X i.e., (X, d) is a multiplicative metric space.

Example1.2. ([10]) Let d: $\mathbb{R} \times \mathbb{R} \to [1, \infty)$ be defined as

$d(x, y) = a^{|x-y|}$,where $x, y \in \mathbb{R}$ and $a > 1$. Then d(x, y) is a multiplicative metric and (X, d) is called a multiplicative metric space. We call it usual multiplicative metric spaces. We note that neither every metric is multiplicative metric nor every multiplicative metric is metric. The mapping d^* defined above is multiplicative metric but not metric as it doesn't satisfy triangular inequality. Consider $d^*(\frac{1}{3}, \frac{1}{2}) + d^*(\frac{1}{2}, 3) = \frac{3}{2} + 6 = 7.5 < 9 = d^*(\frac{1}{3}, 3)$.

On the other, hand the usual metric on R is not multiplicative metric as it doesn't satisfy multiplicative triangular inequality, since $d(2, 3) \cdot d(3, 6) = 3 < 4 = d(2, 6)$.

One can refer to ([8]) for detailed multiplicative metric topology.

Definition1.3.([8]) Let (X, d) be a multiplicative metric space. A sequence $\{x_n\}$ in X said to be a

(i) multiplicative convergent sequence to x, if for every multiplicative open ball $B_\epsilon(x) = \{ y \mid d(x, y) < \epsilon\}$, $\epsilon > 1$, there exists a natural number N such that $x_n \in B_\epsilon(x)$ for all $n \geq N$, i. e, $d(x_n, x) \to 1$ as $n \to \infty$.
(ii) multiplicative Cauchy sequence if for all $\epsilon > 1$, there exists $N \in \mathbb{N}$ such that $d(x_n, x_m) < \epsilon$ for all $m, n > N$ i. e , $d(x_n, x_m) \to 1$ as $n \to \infty$.

A multiplicative metric space is called complete if every multiplicative Cauchy sequence in X is multiplicative converging to $x \in X$.

In 2012, Özavşar and Çevikel[8] introduced the concepts of Banach-contraction, Kannan-contraction, and Chatterjea-contraction mappings in the sense of multiplicative metric spaces as follows:

(Banach-contraction). Let (X, d) be a complete multiplicative metric space and let f: X → X be a multiplicative contraction if there exists a real constant $\lambda \in [0, 1)$ such that

$d(f(x), f(y)) \leq d(x, y)^\lambda$ for all $x, y \in X$. Then f has a unique fixed point.

(Kannan-contraction). Let (X, d) be a complete multiplicative metric space. Suppose the mapping $f : X \to X$ satisfies the contraction condition

$d(fx, fy) \leq (d(fx, x) \cdot d(fy, y))^\lambda$, for all $x, y \in X$, where $\lambda \in [0, \frac{1}{2})$.

Then f has a unique fixed point in X and for any $x \in X$, iterative sequence $(f_n(x))$ converges to the fixed point.

(Chatterjea-contraction). Let (X, d) be a complete multiplicative metric space. Suppose the mapping $f : X \to X$ satisfies the contraction condition

UGC Approved Journal
IARJSET
ISSN (Online) 2393-8021
ISSN (Print) 2394-1588
International Advanced Research Journal in Science, Engineering and Technology
ISO 3297:2007 Certified
Vol. 4, Issue 7, July 2017

$d(fx,fy) \le (d(fy,x) \cdot d(fx,y))^{\lambda}$, for all x, y ∈ X, where $\lambda \in [0, \frac{1}{2})$.
Then f has a unique fixed point in X and for any x ∈ X, iterative sequence $(f_n(x))$ converges to the fixed point.

2. MAIN RESULTS

Theorem 2.1. Let (X, d) be a complete multiplicative metric space. Suppose the mapping
f : X → X be a self- mapping satisfies the following condition:
(2.1) $d(fx,fy) \le [d(x,y)]^{a_1}.[d(x,fy)]^{a_2}.[d(fx,y)]^{a_3}.[d(fy,y)]^{a_4}.[d(fx,x)]^{a_5}$,
for all x, y ∈ X, where $a_1, a_2, a_3, a_4, a_5 \ge 0$ and $a_1 + 2a_2 + 2a_3 + a_4 + a_5 < 1$
Then f has a unique fixed point in X.
Proof. Let$\{x_n\}$be a sequence in X, defined as follows.
Let $x_0 \in X$, $f(x_0) = x_1, f(x_1) = x_2, \cdots, f(x_n) = x_{n+1}, \cdots$.
From (2.1), we have
$d(x_n, x_{n+1}) = d(Tx_{n-1}, Tx_n)$
$\le [d(x_{n-1}, x_n)]^{a_1}.[d(x_{n-1}, fx_n)]^{a_2}.[d(fx_{n-1}, x_n)]^{a_3}.[d(fx_n, x_n)]^{a_4}.[d(fx_{n-1}, x_{n-1})]^{a_5}$
$\le [d(x_{n-1}, x_n)]^{a_1}.[d(x_{n-1}, x_{n+1})]^{a_2}.[d(x_n, x_n)]^{a_3}.[d(x_{n+1}, x_n)]^{a_4}.[d(x_{n-1}, x_n)]^{a_5}$
On simplification, we have
$d(x_n, x_{n+1}) = d(Tx_{n-1}, Tx_n) \le [d(x_{n-1}, x_n)]^{a_1+a_2+a_3+a_5}.\ [d(x_{n+1}, x_n)]^{a_2+a_3+a_4}$
$d(x_n, x_{n+1}) \le [d(x_{n-1}, x_n)]^{h}$,
where $h = \frac{a_1+a_2+a_3+a_5}{1-(a_2+a_3+a_4)} < 1$.
Similarly, $d(x_{n-1}, x_n) \le [d(x_{n-2}, x_{n-1})]^{h}$,
$d(x_n, x_{n+1}) \le [d(x_{n-2}, x_{n-1})]^{h^2}$.
Continue like this we get,
$d(x_n, x_{n+1}) \le [d(x_0, x_1)]^{h^n}$
For n > m, $d(x_n, x_m) \le d(x_n, x_{n-1}) \cdot d(x_{n-1}, x_{n-2}) \cdots d(x_m, x_{m+1})$
$\le d(x_0, x_1)^{\,h^{n-1}+h^{n-2}+\cdots h^m}$
$\le d(x_0, x_1)^{\frac{h^m}{1-h}}$. This implies $d(x_n, x_m) \to 1 (n, m \to \infty)$.
Hence (x_n) is a Cauchy sequence. By the multiplicative completeness of X, there is z ∈ X such that $x_n \to z$ (n →∞).
Now we show that z is fixed point of f .From (2.1), we have
$d(fz, z) \le d(fx_n, fz).\ d(fx_n z)$
$\le [d(z, x_n)]^{a_1}.[d(x_n, fz)]^{a_2}.[d(fx_n, z)]^{a_3}.[d(fz, z)]^{a_4}.[d(fx_n, x_n)]^{a_5}$
$d(fz, z) \le [d(z, fz)]^{a_2+a_4}$ gives fz = z, i.e., z is a fixed point of f.
Uniqueness: Suppose z, w (z ≠ w) be two fixed point of f, then from (2.1), we have
d(z, w) = d(fz, fw)
$\le [d(z,w)]^{a_1}.[d(z,fw)]^{a_2}.[d(fz,w)]^{a_3}.[d(fw,w)]^{a_4}.[d(fz,z)]^{a_5}$
$d(z, w) \le [d(z,w)]^{a_1+a_2+a_3}$ this implies that d(z, w) = 1 i.e., z = w.
Hence f has a unique fixed point .
Corollary 1. Putting $a_2 = a_3 = a_4 = a_5 = 0$ gives Banach-contraction[8].
Corollary 2. Putting $a_1 = a_2 = a_3 = 0$, $a_4 = a_5$ gives Kannan-contraction[8].
Corollary 3. Putting $a_1 = a_4 = a_5 = 0$, $a_2 = a_3$ gives Chatterjea-contraction[8].
Corollary 4. Putting $a_4 = a_5 = 0$, gives Isufati results[5] in the sense of multiplicative metric spaces.
Corollary 5. Putting $a_4 = a_5 = 0$, $a_2 = a_3$ gives Reich results[9] in the sense of multiplicative metric spaces.

REFERENCES

[1] Abbas, M. ,Ali B., Suleiman ,Y. I., Common Fixed Points of Locally Contractive Mappings in Multiplicative Metric Spaces with Application, Hindawi Publishing Corporation International Journal of Mathematics and Mathematical Sciences Volume (2015), Article ID 218683.
[2] Bashirov, A.E., Kurplnara ,E.M., and Ozyapici A., Multiplicative calculus and its applications. J. Math. Anal. Appl. 337, (2008)36-48.
[3] Ciric, L. B. "A generalization of Banach contraction Principle" Proc. Amer. Math. Soc. 25 (1974) 267-273.
[4] Fisher B. "A fixed point theorem for compact metric space" Publ.Inst.Math.25 (1976) 193-194.
[5] Isufati, A., Fixed point theorem in dislocated quasi metric spaces. Appl. Math. Sci.4, (2010) 217-223.
[6] Jaggi, D.S. ,Some unique fixed point theorems. I. J.P. Appl. 8(1977) 223-230.
[7] Kohli, M., Shrivastava, R., Sharma, M., Some results on fixed point theorems in dislocated quasi metric space. Int. J.Theoret. Appl. Sci.2, 27-28 (2010).
[8] Ozavsar ,M. and Cevikel ,A.C., Fixed point of multiplicative contraction mappings on multiplicative metric space, arXiv:1205.5131v1 [matn.GN] (2012).
[9] Reich, S. ,Some remarks concerning contraction mapping. Canada. Math.Bull.14 (1971) 121-124.
[10] Sarwar ,M., Badshah-e ,R., Some unique fixed point Theorems in multiplicative metric space, arXiv:1410.3384v2 [matn.GM] (2014).
[11] Song, M. ,He ,X.and Chen, D., Common fixed points for weak commutative mappings on a multiplicative metric space. Fixed point Theory and Applications (2014), 2014:48.