Adaptive Back stepping Approach for Longitudinal Control of Aircraft Unmanned Aerial Vehicles

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Abstract: Autonomous unmanned aerial vehicles provide the possibility of performing tasks and missions that are currently hazardous or can cost lives. Here a nonlinear model of the unmanned aerial vehicle is used in order to describe the longitudinal dynamics which is highly complicated in itself. Therefore an adaptive back stepping controller is employed, the underlying feature of which is a dynamic parameter adaptation law. The control input is derived based on the Lyapunov stability theorems, that guarantees asymptotic stability design phase itself. The control law along with the parameter adaptation law gives a better regulation when compared with other linear control techniques. Software implementation is carried out using MATHLAB Simulink. Simulation results validate the fact that the controller works satisfactory.

Keywords: Unmanned Aerial Vehicles; Lyapunov stability; Adaptive back stepping controller

I. INTRODUCTION

Unmanned Aerial Vehicles (UAVs) are a viable alternative to manned aircraft and satellites for a variety of applications, including environmental monitoring, agriculture, and surveying. They promise greater precision and much lower operating costs than traditional methods. Critical to the success of unmanned aerial vehicles systems is the auto-pilot system which keeps the vehicle in the air and in control in the absence of a human pilot.

The development of autopilot systems for unmanned aerial vehicles is an area undergoing intense research. The ability to test autopilot systems in a virtual (software) environment using a software flight dynamics model for unmanned Aerial Vehicles is significant for development. A reliable unmanned aerial vehicles simulation process which can be adapted for different aircrafts would provide a platform for developing autopilot systems with reduced dependence on expensive field trials.

Remote sensing, Commercial aerial surveillance, Archaeology, Forest fire detection, Armed attacks, research, Oil, Commercial and Motion Picture Film making, Search and rescue operations, Maritime patrol and Aerial target practice in training of human pilots are the few out of many applications where unmanned aerial vehicles have proved to be an alternative and fill the gap where the risk of human piloted aircraft is not acceptable or impractical. The main building block of unmanned aerial vehicles is its automatic flight controller which is known as autopilot. This subsystem controls unmanned aerial vehicles by generating control signals on the basis of desired target information and waypoints.

Over the past years, various automatic flight control systems for unmanned Aerial Vehicles system have been reported in the literature [1] - [17]. The more autonomous ability of unmanned aerial vehicles, the more complex its guidance and control system, advanced guidance algorithms development is essential and necessary for meeting new requirements with the increasing area of unmanned aerial vehicles applications and for defining future unmanned aerial vehicles concepts and associated critical technologies. The complete state of the unmanned aerial vehicles comprises its position, airspeed (Va), attitudes [roll(θ), pitch(θ), yaw(ψ)] angle-of-attack (α), sideslip angle(β), and rotation (roll (p), pitch(q) and yaw (r)) rates. Position, airspeed, and heading attitude are also known as the navigation states [1]. Control on these states provides full control on the vehicle movements with six degrees of freedom. The requirements of control are to ensure that the dynamics are “fast” and to ensure that the oscillations die out quickly, and also the requirements on a good tracking of command input with minimum steady state errors. Since the open-loop dynamics of the vehicle rarely satisfy these requirements, so the typical approach is to use linear and non-linear feedback control to modify the pole locations and loop gains [2], [3].

With respect to nonlinear control many strategies can be considered in the design as sliding mode controller [4], [5]. In [6], a linear, quadratic regulator method is used to control the trajectory and mission paths of the autonomous unmanned aerial vehicles.
The paper [7] depicts the application of linear quadratic optimal control to the longitudinal flight motion of an unmanned aerial vehicle which has elevon control only. Some advanced techniques including robust and adaptive control techniques are also used in unmanned aerial vehicles control. In paper [8] $H_{\infty}$ technique is used to design a velocity and altitude controller that follows a determined model. In this work an adaptive back stepping controller is designed for the longitudinal controlling of UAVs, whose unknown parametric uncertainties are estimated by using parameter adaptation law. The uncertainties associated with the estimates are not considered. Therefore the estimates are then used as if they are equal to the true parameters. This is called uncertainty equivalence principle. The proposed control law and adaptation laws will ensure asymptotic stability.

II. UAV AIRCRAFT MODEL

The nonlinear equation of motion of the aircraft longitudinal dynamics involve,

II.1 Velocity Dynamics:

\[
\dot{V}_a = \frac{1}{m} \left( -\frac{1}{2} \rho V_a^2 SC_{\rho} + F_T \cos \alpha \cdot \text{msin} \gamma \right)
\]

where $V_a$ is the aerodynamic velocity, $\gamma$ is the flight path angle, $\theta$ is the pitch angle, $q$ is the pitch angular velocity, $F_T$ is the engine thrust, $\delta$ is the elevator angle.

II.2 Pitch Dynamics:

\[
\dot{\gamma} = \frac{1}{m} \left( V_a \sin \alpha - mg \cos \gamma \right)
\]

\[
\theta = q
\]

\[
\dot{q} = \frac{M}{I_y} \delta
\]

where $m$ is the mass, $I_y$ is the moment of inertia, $L$ and $D$ are aerodynamic lift and drag forces, $M(\delta)$ is the pitching moment.

The aerodynamic forces and moment that are computed in terms of non dimensional coefficients in system modeling as,

\[
L = \frac{1}{2} \rho V_a^2 SC_L
\]

\[
D = \frac{1}{2} \rho V_a^2 SC_D
\]

\[
M = \frac{1}{2} \rho V_a^2 \frac{S}{C_a}
\]

where $\rho$ is the air density, $S$ is the reference wing surface, $c$ is the mean chord and $C_L$, $C_D$ and $C_a$ are the lift, drag and pitching moment coefficients. The drag and moment coefficients are as follows:

\[
C_D = C_{D0} + k_1 \alpha + k_2 \alpha^2
\]

\[
C_a = C_{a0} + C_{a\alpha} \alpha + C_{a\gamma} q + C_{a\delta} \delta
\]

where $C_{D0}$, $k_1$, $k_2$, $C_{a0}$, $C_{a\alpha}$, $C_{a\gamma}$, $C_{a\delta}$ are aircraft aerodynamic coefficients.

III. ADAPTIVE BACK STEPPING CONTROLLER DESIGN

Two different controllers has been designed. One for controlling aerodynamic velocity with engine thrust as control input. Second for controlling Flight path angle using elevator deflection angle.

III.1 Control of aerodynamic velocity:

After substituting the equation of drag in the velocity dynamics equation

\[
\dot{V}_a = \frac{1}{m} \left( -\frac{1}{2} \rho V_a^2 SC_{\rho} + F_T \cos \alpha \cdot \text{msin} \gamma \right)
\]

Since the coefficient of lift mainly depends on the angle of attack, the drag model can be considered as follows:

\[
C_D = C_{D0} + k_1 \alpha + k_2 \alpha^2
\]

where $C_{D0}$, $k_1$, $k_2$ are unknown parameters.

Let $V_r$ be the reference velocity and the error in velocity be defined as

\[
z_v = V_a - V_r
\]
Taking first derivative of error variable

\[ \dot{z}_v = V_y \dot{z}_v - V_y \cdot (8) \]

\[ \dot{z}_v = \frac{1}{m} \left( \frac{1}{2} \rho \left( \dot{z}_v + V_y \right)^2 C_{y} + F_y \cos \alpha \cdot \sin \gamma \right) \cdot V_y \]

\[ \dot{z}_v = \beta_1 \left( \dot{z}_v + V_y \right) \cdot \cos \left( \alpha \right) + \frac{F_y \cos \alpha}{m} \cdot \sin \gamma \cdot \dot{V}_y \]

\[ C_D = \varphi \left( \alpha \right)^T \theta \]

where \( \varphi \left( \alpha \right) = \begin{bmatrix} 1 & \alpha & \alpha^2 \end{bmatrix}^T \)

\[ \theta = \begin{bmatrix} C_{D\alpha} & k_1 & k_2 \end{bmatrix}^T \]

\[ \beta_1 = \frac{\rho S}{2m} \]

\[ \gamma = \frac{1}{m} \left( \frac{1}{2} \rho \left( \dot{z}_v + V_y \right)^2 C_{y} + F_y \sin \alpha - mg \cos \gamma \right) \]

\[ q = \frac{\rho \left( \dot{z}_v + V_y \right)^2 \cos \left( \alpha \right)}{2 I_y} \left( C_{m0} + C_{m\alpha} \alpha + C_{mq} q + C_{m\delta} \delta \right) \]

The augmented Lyapunov function given by

\[ W_v = \frac{1}{2} z_v^2 + \frac{1}{2 \Gamma_1} \theta^2 \]

\[ W_v = \frac{1}{2} z_v^2 + \frac{1}{2 \Gamma_1} \theta^2 \quad (9) \]

where error variable defined as

\[ W_v = \dot{z}_v \varphi \left( \alpha \right)^T \theta + \frac{F_y \cos \alpha}{m} \cdot \sin \gamma \cdot \dot{V}_y \]

\[ W_v = \beta_1 \left( \dot{z}_v + V_y \right) \cdot \cos \left( \alpha \right) + \frac{F_y \cos \alpha}{m} \cdot \sin \gamma \cdot \dot{V}_y \]

where

\[ \theta = \theta - \dot{\theta} \quad (16) \]

Denotes the estimation error vector and \( \Gamma_1 \) represent adaptation gain matrix with \( \Gamma_1 > 0 \)

\[ \dot{\theta} = -\beta_1 \left( \dot{z}_v + V_y \right) \Gamma_1 \varphi \left( \alpha \right) \quad (17) \]

The adaptation law and control ensures stability of the system. Since \( W_v \) is positive definite and radially bounded and \( W_v \leq o \), the error velocity and estimated unknown parameter vector is globally bounded and zero.

Control of flight path angle:

After substituting equation (3) in (2) the pitch dynamics of aircraft UAV system becomes

\[ \gamma = \frac{1}{m} \left( \frac{1}{2} \rho \left( \dot{z}_v + V_y \right)^2 C_{y} + F_y \sin \alpha - mg \cos \gamma \right) \]

\[ \theta = \varphi \left( \alpha \right)^T \theta > 0 \quad (11) \]

Considering equation (9), control law obtained as given by

\[ F_{r} = \frac{m}{\cos \alpha} \left( \sin \gamma + \dot{V}_y + \beta_1 \left( \dot{z}_v + V_y \right) \cos \left( \alpha \right) \right) \cdot \varphi \left( \alpha \right)^T \theta - B_{yi} \varphi \left( \alpha \right) \cdot \varphi \left( \alpha \right) \cdot \varphi \left( \alpha \right)^T \theta \quad (12) \]

To design the controller, defining the set of error coordinates as

\[ \dot{z}_3 = \gamma - \gamma_{ref} \quad (19) \]

\[ \dot{z}_2 = \dot{\theta} - \dot{\theta}_{ref} - \alpha \quad (20) \]

\[ \dot{z}_1 = q \quad (21) \]

Taking first derivative in the newest of coordinates

\[ \dot{z}_3 = \eta \left( \dot{z}_3 - \dot{z}_1 \right) \quad (22) \]

\[ \dot{z}_2 = \dot{z}_2 \]

\[ \dot{z}_1 = \beta_1 \left( C_{m0} + C_{m\alpha} \left( \dot{z}_3 - \dot{z}_1 + \alpha \right) + C_{mq} \dot{z}_1 + C_{m\delta} \delta \right) \]
where

\[ \beta = \frac{\rho V^2 S}{2 I_y} \]  

Defining the first Lyapunov function as

\[ f_1 = \frac{1}{2} z_1^2 \]  

(24)

Taking first derivative, (24) becomes

\[ \dot{f}_1 = z_1 \eta (z_2 - z_1) \]  

(25)

In order to make \( f_1 \) negative definite where \( z_2 \) chosen

\[ z_2 = u_1(z_1) = \omega_2 z_1. \]  

For \( f_1 \) to be negative definite \( \omega_2 > -1. \)

Rewriting equation (22) in consideration with error variable \( z_2 = z_2 - u_1(z_1) \) as

\[ \dot{z}_1 = \Omega(\mu) \]  

(26)

\[ \dot{z}_2 = \omega_2 \Omega(\mu) \]  

The Lyapunov function for equation (26)

\[ f_2 = V_1 f_1 + \frac{1}{2} z_2^2 + F(\mu) \]  

(27)

where \( F(\mu) \) is a positive definite function of \( \mu \)

Taking first derivative of

\[ \dot{f}_2 = (\omega_2 V_2 - \omega_1) \mu F(\mu) + F(\mu)^T V_2 + \omega_2 z_2 z_2 ^2 \]  

(28)

In order to make the function negative definite choose

\[ V_1 = \left(1 + \omega_1 \right)(\omega_2 V_2 - \omega_1) \]  

(29)

Defining the third error variable as:

\[ z_3 = z_1 - u_2(z_1, z_2) \]  

(30)

will produce the first derivative of error variable as follows:

\[ \dot{z}_3 = \beta_3 \left(C_{m0} + C_{m\alpha} (\mu + \alpha_0) + C_{mH} z_3 + C_{m\delta} \delta \right) \]

\[ + \omega_2 z_2 \omega_3 + \omega_3 \Omega(\mu) \]  

(31)

For controlling pitch dynamics of the system, here the elevator deflection angle is real control input of aircraft with \( C_{m0}, C_{m\alpha}, C_{mH} \) are taken as the unknown aerodynamic coefficients. For ensuring the stability and to deal with parametric uncertainty an adaptive back stepping controller is designing for equation (31). The first derivative of error variable can be written as:

\[ \dot{z}_3 = \beta_3 \lambda^T \lambda + \beta_3 \delta \dot{\delta} + \omega_2 \left( z_3 - z_2 \omega_2 + \omega_3 \Omega(\mu) \right) \]  

(32)

where

\[ \beta_3 = \frac{\rho V^2 S c}{2 I_y} C_{m\delta}, \]  

(33)

\[ \Lambda = [C_{m0} C_{m\alpha} C_{mH}]^T \]  

(34)

Denotes unknown parameter vector and

\[ \lambda = [1 \quad \mu + \alpha_0 \quad z_3 \omega_2]^T \]  

(35)

The augmented Lyapunov function in consideration with error variable as follows:

\[ \dot{f}_3 = V_3 f_3 + \frac{1}{2} z_3^2 + \frac{1}{2} \lambda^T \Lambda^T \]  

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Taking the first derivative of Lyapunov function

\[ f_z = -V \left[ (a_0, a_0, a_0) \mu \cdot \nabla \mu + a_0 \cdot a_0 \cdot z \cdot \nabla \mu \right] + \alpha \sum_{i} a_i \cdot z \cdot \nabla \mu \] \tag{37}

where \( \Gamma_2 \) is the adaptation gain matrix such that \( \Gamma_2 > 0 \) and the estimation error vector be represented as

\[ \tilde{z} = z - \hat{z} \] \tag{38}

In order to make the first derivative of function negative definite so as to ensure asymptotic stability, consider terms involving \( \Omega \left( \mu \right) \) such that

\[ -V_{z} V_{z} \Omega \left( \mu \right) + (V_{z} V_{z} + \omega_{2} \omega_{1}) \tilde{z} z \Omega \left( \mu \right) \]

\[ = \left( \sqrt{V_{z} V_{z} \Omega \left( \mu \right)} - \sigma \tilde{z} \right)^2 + \sigma^2 \tilde{z}^2 \] \tag{39}

Thus the first derivative of augmented Lyapunov function can be rewritten as:

\[ f_z = -V \left[ (a_0, a_0, a_0) \mu \cdot \nabla \mu + a_0 \cdot a_0 \cdot z \cdot \nabla \mu \right] + \alpha \sum_{i} a_i \cdot z \cdot \nabla \mu \] \tag{41}

Thus the control and adaptation law is obtained as

\[ \delta_z = \frac{1}{\beta_{\delta}} \left( -\omega_{2} \tilde{z} \right) \] \tag{42}

\[ \Lambda = -\Lambda = \beta_2 \Gamma_2 \tilde{z} \] \tag{43}

On substitution in equation (42), Control law for pitch dynamics with elevator deflection as control input becomes as follows:

\[ \delta_z = \frac{1}{\beta_{\delta}} \left( -\omega_{2} \tilde{z} \right) \] \tag{44}

On substitution in equation (43), the parametric updating law concerning pitch dynamics for dealing with unknown parametric uncertainty is as given below:

\[ \Lambda = \beta_2 (q + \alpha_2 (\theta - \gamma_{ref} - \alpha_{1} + \alpha_{1} (\gamma - \gamma_{ref}))) \beta_2 \Gamma_2 \] \tag{45}

IV. SIMULATION RESULTS

In this section, the results of numerical simulation are presented in order to demonstrate the performance of the proposed adaptive back stepping controller. Parameters used for the simulation is given in the table 1.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Value</th>
<th>units</th>
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<tr>
<td>m</td>
<td>Aircraft Mass</td>
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<td>Kg</td>
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<tr>
<td>Q</td>
<td>Dynamic pressure</td>
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<td>S</td>
<td>Wing area</td>
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<td>m²</td>
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<td>I_s</td>
<td>Moment of inertia around x-axis</td>
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<td>Kg.m²</td>
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<tr>
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<td>Kg.m²</td>
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<tr>
<td>I_z</td>
<td>Moment of inertia around z-axis</td>
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<td>Kg.m²</td>
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<td>( \alpha )</td>
<td>Mean chord length</td>
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<tr>
<td>g</td>
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<td>m_w</td>
<td>Empty weight</td>
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<tr>
<td>aR</td>
<td>Aspect ratio</td>
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<td></td>
</tr>
<tr>
<td>V_a</td>
<td>Aircraft velocity</td>
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<td>m/s</td>
</tr>
</tbody>
</table>

Table 1: UAV model parameters

The simulation of velocity variation with respect to time with initial condition 1 is shown in fig 2. The fig 2 shows regulation curve. The simulation is done for \( \Lambda = [-1, -1, -10] \). The results shows that the controller enforces the system to eliminate disturbance and provides better regulation. System is converging to zero even in the presence of uncertainty.
Fig 2: Variation of velocity with respect to time

The simulation of velocity variation with respect to time with initial condition .01 is shown in fig 3. The simulation is done for \( \Lambda = [-.1 -1 -10] \). The results show that the controller enforces the system to eliminate disturbance and is converging to zero even in the presence of uncertainty.

Fig 3: Variation of velocity with respect to time

The simulation results shown in fig 4 and fig 5 represent the tracking behavior of the compensated system with different initial conditions. The simulation is done for \( \Lambda = [-.1 -1 -10] \). By using adaptive back stepping control law and parametric updating law, the UAV system can track a reference velocity even in the presence of external disturbance and unknown parametric uncertainties. Ensures satisfactory performance with the incorporation of proposed controller.

Fig 4: Variation of velocity with respect to time

Fig 5: Variation of velocity with respect to time

The simulation of flight path angle with respect to time is shown in figure 6 and 7. Both the figures show the

Fig 6: Variation of flight path angle with respect to time

Fig 7: Variation of flight path angle with respect to time
regulation curves. Better regulation of the longitudinal pitch dynamics is ensued by the incorporation of the adaptive back stepping controller. The simulation is done with parameter values \( \theta = [0.05 \ 0.05 \ 0.05] \). The results shows better convergence of the system against parametric uncertainties even in the presence of external disturbances. The fig 6 shows regulation curve of pitch dynamics with initial condition .15 and the fig 7 shows the regulation curve with initial condition .3. Both aids the system to converge the origin.

V. CONCLUSIONS

In order to stabilize the nonlinear equations of motion concerning longitudinal dynamics of the aircraft UAV, an adaptive back stepping approach is employed. Lyapunov stability theorem is used to verify the stability of the system. Asymptotic stability is ensured by the theorem in the design procedure. Simulation results for velocity control ensure both tracking and regulation with proposed controller design. Simulation results for pitch angle control ensure good regulation.

REFERENCES