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Cubic Lateral Ideals in Ternary Near-Rings

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Abstract: In this paper, we introduced the notion of cubic lateral ideals in ternary near-rings and obtain some characterizations of cubic lateral ideals in ternary near-rings. Finally, we investigate some related properties using the concepts of cubic homomorphism and anti-homomorphism between ternary near-rings.

Keywords: Ternary near-rings, lateral ideal, cubic lateral ideal, cubic homomorphism.

I. INTRODUCTION

Zadeh [16,17] in 1965 introduced the notion of fuzzy set. The concept of fuzzy subgroup was first introduced by Rosenfeld [11] in 1971. In 1991, Abou-Zaid [2] investigated the ideal of fuzzy subnear-rings and fuzzy ideals in near-rings. Kim et al [8] applied a few concepts of fuzzy ideals of near-rings. Ternary near-ring is the generalized structure of near-ring. Lehmer [9] in 1932 introduced the notion of ternary algebraic system. Dutta et al [5] introduced the concept of ternary semi ring which is a generalization of the ternary ring introduced by Lister [10]. To discuss these results to near-ring using ternary product Warud Nakkhaseen et al [15] have applied the ideal of ternary semi ring to define left ternary near-ring, ternary subnear-ring and their ideals. Thillaigovindan et al [13] discussed the concept of Interval valued fuzzy ideals of near-rings. Jun et al [7] introduced the concept of cubic sets. This structure encompasses interval-valued fuzzy set and fuzzy set. Also Jun et al [6] introduced the notion of cubic subgroups. Chinnadurai et al [3] introduced the notion of cubic bi-ideals in near-rings.

The purpose of this paper to introduce the notion of cubic lateral ideals in ternary near-rings and concept of homomorphism and anti- homomorphism between ternary near-rings**.** We Investigate some basic results, properties and examples.

II.PRELIMINARIES

In this section, we present some definitions that are used in the sequel.

Definition 2.1 [12] Let R be a non-empty set and [ ] be an operation defined from $R \times R \times R$ to R called a ternary operation. Then $(R, [\,])$ is a ternary semigroup if for every $x, y, z, u, v \in R$,

$\big[[xyz]uv\big] = [x[yzu]v] = \big[xy[zuv]\big]$.

Definition 2.2 [12] Let A, B, C be non-empty subsets of a ternary semi group R. Then $[ABC] = \{[abc] \in R \,\vdots\, a \in A, b \in B, c \in C\}$.

Definition 2.3 [14] Let R be a non-empty set together with a binary operation + and ternary operation $[\,]: R \times R \times R$ to R. Then $(R, +, [\,])$ is a right (left, lateral) ternary near-ring if

i) $(R, +)$ is a group (not necessarily abelian)

ii) $(R, [\,])$ is a ternary semi group

iii) $[(a + b)cd] = [acd] + [bcd]$,

$([cd(a + b)] = [cda] + [cdb], [c(a + b)d] = [cad] + [cbd])$

for every $a, b, c, d \in R$.

Throughout this paper, R denotes a right ternary near-ring.

Definition 2.4. [15] A non-empty subset S of a ternary near-ring R is called a ternary subnear-ring of R, if

i) $x - y \in S \quad \forall x, y \in S$

ii) $[SSS] \subseteq S$.

Definition 2.5 [14] Let R be a right ternary near-ring, Let $(I, +)$ be a normal subgroup of $(R, +)$. Then I is called (i) a right ideal of R if $[IRR] \subseteq I$, (ii) a left ideal of R]if $[xy(z + c)z] - [xyz] \in I$, (iii) a lateral ideal of R if $[x(y + c)z] - [xyz] \in I$ where $x, y, z \in R, c \in I$. I is an ideal of R if it is a right, lateral and left ideal of R.

Definition 2.6. [4] Let I be an lateral ideal of R. For each $a + I, b + I$ in the factor group R/I, we define $(a + I) + (b + I) = (a + b) + I$ and $(a + I)(b + I) = (ab) + I$. Then R/I is a near-ring which we call the residue class near-ring of R with respect to I.

Definition 2.7. [1] A mapping $\mu: X \to [0,1]$ is called a fuzzy subset of X.

Definition 2.8. [4] A fuzzy subset μ of R is called a fuzzy subternary near-ring of R if

i) $\mu(x - y) \geq \min\{\mu(x), \mu(y)\}$

ii) $\mu([xyz]) \geq \min\{\mu(x), \mu(y), \mu(z)\}$

for all $x, y, z \in R$.

Definition 2.9. [4] A fuzzy subset μ of R is called a fuzzy ideal of R if

i) $\mu(x - y) \geq \min\{\mu(x), \mu(y)\}$

ii) $\mu(y + x - y) \geq \mu(x)$

iii) $\mu([xyz]) \geq \mu(x)$

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iv) $\mu([xy(z+c)]-[xyz]) \geq \mu(c)$
v) $\mu([x(y+c)z]-[xyz]) \geq \mu(c)$
for every $x, y, z, c \in R$. If μ is a fuzzy left ideal of R if it satisfies (i), (ii) and (iv). If μ is a fuzzy right ideal of R if it satisfies (i), (ii) and (iii). If μ is a fuzzy lateral ideal of R if it satisfies (i), (ii) and (v).

Definition 2.10. [1] Let X be a non-empty set. A mapping $\bar{\mu}: X \rightarrow D[0,1]$ is called interval-valued fuzzy set, where D[0,1] denote the family of all closed sub intervals of [0,1] and $\bar{\mu}(x)=[\mu^{-}(x), \mu^{+}(x)]$ for all $x \in X$, where μ^{-} and μ^{+} are fuzzy subsets of X such that $\mu^{-}(x) \leq \mu^{+}(x)$ for all $x \in X$.

Definition 2.11. [6] Let X be a non-empty set. A cubic set $\mathcal{A}$ in X is a structure $\mathcal{A}=\{\langle x, \bar{\mu}_A(x), f_A(x)\rangle : x \in X\}$ which is briefly denoted by $\mathcal{A}=\langle\bar{\mu}_A, f_A\rangle$, where $\bar{\mu}_A=[\mu_A^{-}, \mu_A^{+}]$ is an interval-valued fuzzy set (briefly, IVF) in X and f is a fuzzy set in X. In this case, we will use $\mathcal{A}(x)=\langle\bar{\mu}_A(x), f_A(x)\rangle$

$$= \langle[\mu^{-}(x), \mu^{+}(x)], f_A(x)\rangle \;\forall\, x \in X.$$

Definition 2.12. [7] For any non-empty subset G of a set X, the characteristic cubic set of G is defined to be a structure $\chi_G(x) =< x, \bar{\mu}_{\chi_G}(x), \gamma_{\chi_G}(x) : x \in X >$ which is briefly denoted by $\chi_G(x) =< \bar{\mu}_{\chi_G}(x), \gamma_{\chi_G}(x) >$ where

$$\bar{\mu}_{\chi_G}(x)=\begin{cases}[1,1] & \text{if } x \in G\\ [0,0] & \text{otherwise}\end{cases}$$ and

$$\gamma_{\chi_G}(x)=\begin{cases}0 & \text{if } x \in G\\ 1 & \text{otherwise}\end{cases}$$

III. MAIN RESULTS

In this section we define cubic lateral ideals in right ternary near-rings and obtain some characterizations of cubic lateral ideals in right ternary near-rings.

Definition 3.1. A cubic set $\mathcal{A}=\langle\bar{\mu}, \omega\rangle$ of R is called a cubic subternary near-ring of R if

i) $\bar{\mu}(x-y) \geq \min\{\bar{\mu}(x), \bar{\mu}(y)\}$ and
$\omega(x-y) \leq \max\{\omega(x), \omega(y)\}$

ii) $\bar{\mu}([xyz]) \geq \min\{\bar{\mu}(x), \bar{\mu}(y), \bar{\mu}(z)\}$ and
$\omega([xyz]) \leq \max\{\omega(x), \omega(y), \omega(z)\}$ for all $x, y, z \in R$.

Definition 3.2. A cubic set $\mathcal{A}=\langle\bar{\mu}, \omega\rangle$ of R is called a cubic ideal of R if

i) $\bar{\mu}(x-y) \geq \min\{\bar{\mu}(x), \bar{\mu}(y)\}$ and
$\omega(x-y) \leq \max\{\omega(x), \omega(y)\}$

ii) $\bar{\mu}(y+x-y) \geq \bar{\mu}(x)$ and
$\omega(y+x-y) \leq \omega(x)$

iii) $\bar{\mu}([xyz]) \geq \bar{\mu}(x)$ and
$\omega([xyz]) \leq \omega(x)$

iv) $\bar{\mu}([xy(z+c)]-[xyz]) \geq \bar{\mu}(c)$ and
$\omega([xy(z+c)]-[xyz]) \leq \omega(c)$

v) $\bar{\mu}([x(y+c)z]-[xyz]) \geq \bar{\mu}(c)$ and
$\omega([x(y+c)z]-[xyz]) \leq \omega(c)$

for every $x, y, z, c \in R$. The cubic set $\mathcal{A}=\langle\bar{\mu}, \omega\rangle$ is a cubic left ideal of R if it satisfies (i), (ii) and (iv). The cubic set $\mathcal{A}=\langle\bar{\mu}, \omega\rangle$ is a cubic right ideal of R if it satisfies (i), (ii) and (iii). The cubic set $\mathcal{A}=\langle\bar{\mu}, \omega\rangle$ is a cubic lateral ideal of R if it satisfies (i), (ii) and (v).

Example 3.3. Let $R=\{a, b, c, d\}$ be a set with two binary operations defined as follows

+	a	b	c	d
a	a	b	c	d
b	b	a	d	c
c	c	d	b	a
d	d	c	a	b

·	a	b	c	d
a	a	a	a	a
b	a	a	a	a
c	a	a	a	c
d	a	a	a	d

Define the ternary product [ ] of R by $[xyz]=(x \cdot y) \cdot z$ for every $x, y, z \in R$. Then $(R, +, [\,])$ is a right ternary near-ring.

Define a cubic set $\mathcal{A}=\langle\bar{\mu}, \omega\rangle$ in R by $\bar{\mu}(a)=[0.8,0.9]$, $\bar{\mu}(b)=[0.5,0.6]$, $\bar{\mu}(c)=[0.2,0.3]=\bar{\mu}(d)$ is an interval-valued fuzzy lateral ideal of R and $\omega(a)=0.2$, $\omega(b)=0.3, \omega(c)=0.7=\omega(d)$ is a fuzzy lateral ideal of R. Thus $\mathcal{A}=\langle\bar{\mu}, \omega\rangle$ is a cubic lateral ideal of R.

Definition 3.4. Let $\mathcal{A}_i=\langle\bar{\mu}_i, \omega_i\rangle$ be cubic lateral ideals of near-rings R_i for $i=1,2,3, \ldots, n$. Then the cubic direct product of $\mathcal{A}_i,(i=1,2,3, \ldots, n)$ is a function

$$(\bar{\mu}_1 \times \bar{\mu}_2 \times, \ldots, \times \bar{\mu}_n):(R_1 \times R_2 \times \ldots \times R_n) \rightarrow D[0,1] \text{ and}$$

$$(\omega_1 \times \omega_2 \times, \ldots, \times \omega_n): R_1 \times R_2 \times \ldots \times R_n \rightarrow [0,1]$$ defined by

$$(\bar{\mu}_1 \times \bar{\mu}_2 \times, \ldots, \times \bar{\mu}_n)(x_1, x_2, \ldots, x_n) = \min\{\bar{\mu}_1(x_1), \bar{\mu}_2(x_2), \ldots, \bar{\mu}_n(x_n)\}$$ and

$$(\omega_1 \times \omega_2 \times, \ldots, \times \omega_n)(x_1, x_2, \ldots, x_n) = \max\{\omega_1(x_1), \omega_2(x_2), \ldots, \omega_n(x_n)\}.$$

Lemma 3.5. Let $\mathcal{A}=\langle\bar{\mu}, \omega\rangle$ be a cubic lateral ideal of R. If $\mathcal{A}(x) \sqsubset \mathcal{A}(y)$ that is $\bar{\mu}(x)<\bar{\mu}(y)$ and $\omega(x)>\omega(y)$ then $\bar{\mu}(x-y)=\bar{\mu}(x)=\bar{\mu}(y-x)$ and $\omega(x-y)=\omega(x)=\omega(y-x)$

Proof: Let $\mathcal{A}=\langle\bar{\mu}, \omega\rangle$ be a cubic lateral ideal of R. Let $x, y \in R$. Then

$$\bar{\mu}(x-y) \geq \min\{\bar{\mu}(x), \bar{\mu}(y)\}=\bar{\mu}(x)$$
$$\begin{aligned}\bar{\mu}(x) &= \bar{\mu}(x-y+y)\\ &= \bar{\mu}((x-y)-(-y))\\ &= \bar{\mu}((x-y)-(y))\\ &\geq \min\{\bar{\mu}(x-y), \bar{\mu}(y)\}\\ &= \bar{\mu}(x-y)\end{aligned}$$ again

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$\bar{\mu}(y-x) \geq min\{\bar{\mu}(y), \bar{\mu}(x)\} = \bar{\mu}(x)$
$\bar{\mu}(x) = \bar{\mu}(x-y+y)$
$= \bar{\mu}(y-(y-x))$
$\geq min\{\bar{\mu}(y), \bar{\mu}(y-x)\}$
$= \bar{\mu}(y-x)$.
Similarly we can prove the other result.

Theorem 3.6. If $\mathcal{A} = \langle \bar{\mu}, \omega \rangle$ is a cubic lateral ideal of R, then the set $R_{\mathcal{A}} = \{x \in R \mid \mathcal{A}(x) = \mathcal{A}(0)\}$ is a lateral ideal of R.

Proof: Since $\mathcal{A} = \langle \bar{\mu}, \omega \rangle$ be a cubic lateral ideal of R and $x, y \in R$, then $\mathcal{A}(x) = \mathcal{A}(0)$ and $\mathcal{A}(y) = \mathcal{A}(0)$. Suppose $x, y, z \in R_{\mathcal{A}}$.
Then $\bar{\mu}(x) = \bar{\mu}(y) = \bar{\mu}(z) = \bar{\mu}(c) = \bar{\mu}(0)$ and $\omega(x) = \omega(y) = \omega(z) = \omega(c) = \omega(0)$
Since, $\bar{\mu}$ is an i-v fuzzy lateral ideal of R $\bar{\mu}(x-y) \geq min\{\bar{\mu}(x), \bar{\mu}(y)\} = min\{\bar{\mu}(0), \bar{\mu}(0)\} = \bar{\mu}(0)$
and ω is a fuzzy lateral ideal of R $\omega(x-y) \leq max\{\omega(x), \omega(y)\} = max\{\omega(0), \omega(0)\} = \omega(0)$
Thus $x - y \in R_{\mathcal{A}}$ .For every $y \in R$ and $x \in R_{\mathcal{A}}$ we have $\bar{\mu}(y+x-y) \geq \bar{\mu}(x) = \bar{\mu}(0)and\omega(y+x-y) \leq \omega(x) = \omega(0)$
Thus $y + x - y \in R_{\mathcal{A}}$.
Let $x, y, z \in R$ and $c \in R_{\mathcal{A}}$
$\bar{\mu}([x(y+c)z] - [xyz]) \geq \bar{\mu}(c) = \bar{\mu}(0)$ $\omega([x(y+c)z]-[xyz]) \leq \omega(c) = \omega(0)$
Thus $[x(y+c)z] - [xyz] \in R_{\mathcal{A}}$.
Therefore, $R_{\mathcal{A}}$is a lateral ideal of R.

Theorem 3.7. Let I be a lateral ideal of R. If $\mathcal{A} = <\bar{\mu}, \omega>$ is a cubic lateral ideal of R, then the cubic set $\mathcal{A} = <\bar{\mu}, \omega>$ of R/I defined by $\bar{\mu}(a+I) = \sup_{x \in I} \bar{\mu}(a+x)$ and $\omega(a+I) = \inf_{x \in I} \omega(a+x)$ for all $x \in I$ is a cubic lateral ideal of the residue class ternary near-ring R/I of R with respect to I.

Proof: Let $a, b \in R$ be such that $a + I = b + I$. Then $b = a + y$ for some $y \in I$.
$\bar{\mu}(b+I) = \sup_{x \in I} \bar{\mu}(b+x)$
$= \sup_{x \in I} \bar{\mu}(a+y+x)$
$= \sup_{x+y=i \in I} \bar{\mu}(a+i)$
$= \bar{\mu}(a+I)$
$\omega(b+I) = \inf_{x \in I} \omega(b+x)$
$= \inf_{x \in I} \omega(a+y+x)$
$= \inf_{x+y=i \in I} \omega(a+i)$
$= \omega(a+I)$
This means that $\mathcal{A} = <\bar{\mu}, \omega>$ is well defined. Let $x+I, y+I, z+I, c+I \in R/I$
$\bar{\mu}((x+I)-(y+I)) = \bar{\mu}((x-y)+I)$
$= \sup_{i \in I} \bar{\mu}((x-y)+i)$
$= \sup_{i=p-q \in I} \bar{\mu}((x-y)+(p-q))$
$= \sup_{p,q \in I} \bar{\mu}((x+p)-(y+q))$
$\geq \sup_{p,q \in I} min\{\bar{\mu}(x+p), \bar{\mu}(y+q)\}$
$= min\{\sup_{p \in I} \bar{\mu}(x+p), \sup_{q \in I} \bar{\mu}(y+q)\}$
$= min\{\bar{\mu}(x+I), \bar{\mu}(y+I)\}$
$\omega((x+I)-(y+I)) = \omega((x-y)+I)$
$= \inf_{i \in I} \omega((x-y)+i)$
$= \inf_{i=p-q \in I} \omega((x-y)+(p-q))$
$= \inf_{p,q \in I} \omega((x+p)-(y+q))$
$\leq \inf_{p,q \in I} max\{\omega(x+p), \omega(y+q)\}$
$= max\{\inf_{p \in I} \omega(x+p), \inf_{q \in I} \omega(y+q)\}$
$= max\{\omega(x+I), \omega(y+I)\}$
$\bar{\mu}((y+I)+(x+I)-(y+I))$
$= \bar{\mu}((y+x-y)+I)$
$= \sup_{i \in I} \bar{\mu}((y+x-y)+i)$
$= \sup_{i=q+p-q \in I} \bar{\mu}((y+x-y)+(q+p-q))$
$= \sup_{p \in I, q \in I} \bar{\mu}((y+q)+(x+p)-(y+q))$
$\geq \sup_{p \in I} \bar{\mu}(x+p)$
$= \bar{\mu}(x+I)$
$\omega((y+I)+(x+I)-(y+I))$
$= \omega((y+x-y)+I)$
$= \inf_{i \in I} \omega((y+x-y)+i)$
$= \inf_{i=q+p-q \in I} \omega((y+x-y)+(q+p-q))$
$= \inf_{p \in I, q \in I} \omega((y+q)+(x+p)-(y+q))$
$\leq \inf_{p \in I} \omega(x+p)$
$= \omega(x+I)$
$\bar{\mu}([(x+I)((y+I)+(c+I))(z+I)] - [(x+I)(y+I)(z+I)])$
$= \bar{\mu}(([x(y+c)z]-[xyz])+I)$
$= \sup_{i \in I} \bar{\mu}(([x(y+c)z]-[xyz])+i)$
$\geq \sup_{i \in I} \bar{\mu}([xcz]+[xiz])$ since $xiz \in I$
$= \sup_{i \in I} \bar{\mu}([x(c+i)z])$
$\geq \sup_{i \in I} \bar{\mu}(c+i)$
$= \bar{\mu}(c+I)$
$\omega([(x+I)((y+I)+(c+I))(z+I)] - [(x+I)(y+I)(z+I)])$
$= \omega(([x(y+c)z]-[xyz])+I)$
$= \inf_{i \in I} \omega(([x(y+c)z]-[xyz])+i)$
$\leq \inf_{i \in I} \omega([xcz]+[xiz])$ since $xiz \in I$
$= \inf_{i \in I} \omega([x(c+i)z])$
$\leq \inf_{i \in I} \omega(c+i)$
$= \omega(c+I)$
Hence, $\mathcal{A} = <\bar{\mu}, \omega>$ is a cubic lateral ideal of R/I.

Theorem 3.8. If $\{\mathcal{A}_i\} = <\bar{\mu}_i, \omega_i | i \in \wedge>$ be a family of cubic lateral ideal of R, then the cubic set $\sqcap_{i \in \wedge} \mathcal{A}_i = <\cap_{i \in \wedge} \bar{\mu}_i, \cup_{i \in \wedge} \omega_i>$ is also a cubic lateral ideal of R, where $\wedge$ is any index set.

Proof: Let $\mathcal{A}_i = <\bar{\mu}_i, \omega_i | i \in \wedge>$ be a family of cubic lateral ideals of R.
Let $x, y, z, c \in R$ and $\bar{\mu} = \cap \bar{\mu}_i$; $\omega = \cup \omega_i$ then
$\bar{\mu}(x) = \cap \bar{\mu}_i(x) = (inf\ \bar{\mu}_i)(x)$
$= inf\ \bar{\mu}_i(x)$

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$$\omega(x) = \cup\, \omega_i(x) = (sup\ \omega_i)(x) = sup\ \omega_i(x)$$

$$\bar{\mu}(x-y) = inf\ \bar{\mu}_i(x-y) \geq inf\ \{min\{\bar{\mu}_i(x), \bar{\mu}_i(y)\}\} = min\{inf\ \bar{\mu}_i(x), inf\ \bar{\mu}_i(y)\} = min\{\cap\, \bar{\mu}_i(x), \cap\, \bar{\mu}_i(y)\}$$

and

$$\omega(x-y) = sup\ \omega_i(x-y) \leq sup\{max\{\omega_i(x), \omega_i(y)\}\} = max\ \{sup\ \omega_i(x),\ sup\ \omega_i(y)\} = max\ \{\cup\, \omega_i(x), \cup\, \omega_i(y)\}$$

$$\omega(x-y) = max\{\omega(x), \omega(y)\}$$

$$\bar{\mu}(y+x-y) = inf\{\bar{\mu}_i\ (y+x-y)\} \geq inf\{\bar{\mu}_i(x)\} = \cap\, \bar{\mu}_i(x) = \bar{\mu}(x)$$

$$\omega(y+x-y) = sup\{\omega_i\ (y+x-y)\} \leq sup\{\omega_i(x)\} = \cup\, \omega_i(x) = \omega(x)$$

$$\bar{\mu}([x(y+c)z]-[xyz]) = inf\ \{\bar{\mu}_i\ ([x(y+c)z]-[xyz])\} \geq inf\{\bar{\mu}_i(c)\} = \cap\, \bar{\mu}_i(c) = \bar{\mu}(c)$$

$$\omega([x(y+c)z]-[xyz]) = sup\{\omega_i\ ([x(y+c)z]-[xyz])\} \leq sup\{\omega_i(c)\} = \cup\, \omega_i(c) = \omega(c)$$

Hence, $\sqcap_{i\in\wedge} \mathcal{A}_i = < \cap_{i\in\wedge} \bar{\mu}_i, \cup_{i\in\wedge} \omega_i >$ is a cubic lateral ideal of R.

Theorem 3.9. The direct product of cubic lateral ideals of ternary near-rings is also a cubic lateral ideal of ternary near-ring.

Proof: Let $\mathcal{A}_i = \langle \bar{\mu}_i, \omega_i \rangle$ be cubic lateral ideals of ternary near-rings R_i for $i = 1,2,3,\dots,n$. Let $x = (x_1, x_2, \dots, x_n)$, $y = (y_1, y_2, \dots, y_n)$, $z = (z_1, z_2, \dots, z_n), c = (c_1, c_2, \dots, c_n) \in R_1 \times R_2 \times \dots \times R_n$.

$$\begin{aligned}\bar{\mu}_i(x-y) &= \bar{\mu}_i((x_1, x_2, \dots, x_n) - (y_1, y_2, \dots, y_n)) \\ &= \bar{\mu}_i(x_1 - y_1, x_2 - y_2, \dots, x_n - y_n) \\ &= min\{\bar{\mu}_1(x_1 - y_1), \bar{\mu}_2(x_2 - y_2), \dots, \bar{\mu}_n(x_n - y_n)\} \\ &\geq min\{min\{\bar{\mu}_1(x_1), \bar{\mu}_1(y_1)\}, min\{\bar{\mu}_2(x_2), \bar{\mu}_2(y_2)\}, \dots, min\{\bar{\mu}_n(x_n), \bar{\mu}_n(y_n)\}\} \\ &= min\{min\{\bar{\mu}_1(x_1), \bar{\mu}_2(x_2), \dots, \bar{\mu}_n(x_n)\}, min\{\bar{\mu}_1(y_1), \bar{\mu}_2(y_2), \dots, \bar{\mu}_n(y_n)\}\} \\ &= min\{(\bar{\mu}_1 \times \bar{\mu}_2 \times, \dots, \times \bar{\mu}_n)(x_1, x_2, \dots, x_n), (\bar{\mu}_1 \times \bar{\mu}_2 \times, \dots, \times \bar{\mu}_n)(y_1, y_2, \dots, y_n)\}\end{aligned}$$

$$\bar{\mu}_i(x-y) = min\{\bar{\mu}_i(x), \bar{\mu}_i(y)\}$$

and

$$\begin{aligned}\omega_i(x-y) &= \omega_i((x_1, x_2, \dots, x_n) - (y_1, y_2, \dots, y_n)) \\ &= \omega_i(x_1 - y_1, x_2 - y_2, \dots, x_n - y_n) \\ &= max\{\omega_1(x_1 - y_1), \omega_2(x_2 - y_2), \dots, \omega_n(x_n - y_n)\} \\ &\leq max\{max\{\omega_1(x_1), \omega_1(y_1)\}, max\{\omega_2(x_2), \omega_2(y_2)\}, \dots, max\{\omega_n(x_n), \omega_n(y_n)\}\} \\ &= max\{max\{\omega_1(x_1), \omega_2(x_2), \dots, \omega_n(x_n)\}, max\{\omega_1(y_1), \omega_2(y_2), \dots, \omega_n(y_n)\}\} \\ &= max\{(\omega_1 \times \omega_2 \times, \dots, \times \omega_n)(x_1, x_2, \dots, x_n), (\omega_1 \times \omega_2 \times, \dots, \times \omega_n)(y_1, y_2, \dots, y_n)\}\end{aligned}$$

$$\omega_i(x-y) = max\{\omega_i(x), \omega_i(y)\}$$

$$\begin{aligned}\bar{\mu}_i(y+x-y) &= \bar{\mu}_i((y_1, y_2, \dots, y_n) + (x_1, x_2, \dots, x_n) - (y_1, y_2, \dots, y_n)) \\ &= \bar{\mu}_i(y_1 + x_1 - y_1, y_2 + x_2 - y_2, \dots, y_n + x_n - y_n) \\ &= min\{\bar{\mu}_1(y_1 + x_1 - y_1), \bar{\mu}_2(y_2 + x_2 - y_2), \dots, \bar{\mu}_n(y_n + x_n - y_n)\} \\ &\geq min\{\bar{\mu}_1(x_1), \bar{\mu}_2(x_2), \dots, \bar{\mu}_n(x_n)\} \\ &= (\bar{\mu}_1 \times \bar{\mu}_2 \times, \dots, \times \bar{\mu}_n)(x_1, x_2, \dots, x_n) \\ &= \bar{\mu}_i(x)\end{aligned}$$

$$\begin{aligned}\omega_i(y+x-y) &= \omega_i((y_1, y_2, \dots, y_n) + (x_1, x_2, \dots, x_n) - (y_1, y_2, \dots, y_n)) \\ &= \omega_i(y_1 + x_1 - y_1, y_2 + x_2 - y_2, \dots, y_n + x_n - y_n) \\ &= max\{\omega_1(y_1 + x_1 - y_1), \omega_2(y_2 + x_2 - y_2), \dots, \omega_n(y_n + x_n - y_n)\} \\ &\leq max\{\omega_1(x_1), \omega_2(x_2), \dots, \omega_n(x_n)\} \\ &= (\omega_1 \times \omega_2 \times, \dots, \times \omega_n)(x_1, x_2, \dots, x_n) \\ &= \omega_i(x)\end{aligned}$$

$$\begin{aligned}\bar{\mu}_i([x(y+c)z]-[xyz]) &= \bar{\mu}_i([(x_1, x_2, \dots, x_n)((y_1, y_2, \dots, y_n) + (c_1, c_2, \dots, c_n))(z_1, z_2, \dots, z_n)] - [(x_1, x_2, \dots, x_n)(y_1, y_2, \dots, y_n)(z_1, z_2, \dots, z_n)]) \\ &= \bar{\mu}_i([x_1(y_1 + c_1)z_1] - [x_1 y_1 z_1], [x_2(y_2 + c_2)z_2] - [x_2 y_2 z_2], \dots, [x_n(y_n + c_n)z_n] - [x_n y_n z_n]) \\ &= min\{\bar{\mu}_1([x_1(y_1 + c_1)z_1] - [x_1 y_1 z_1]), \bar{\mu}_2([x_2(y_2 + c_2)z_2] - [x_2 y_2 z_2]), \dots, \bar{\mu}_n([x_n(y_n + c_n)z_n] - [x_n y_n z_n])\} \\ &\geq min\{\bar{\mu}_1(c_1), \bar{\mu}_2(c_2), \dots, \bar{\mu}_n(c_n)\} \\ &= (\bar{\mu}_1 \times \bar{\mu}_2 \times, \dots, \times \bar{\mu}_n)(c_1, c_2, \dots, c_n) \\ &= \bar{\mu}_i(c)\end{aligned}$$

$$\begin{aligned}\omega_i([x(y+c)z]-[xyz]) &= \omega_i([(x_1, x_2, \dots, x_n)((y_1, y_2, \dots, y_n) + (c_1, c_2, \dots, c_n))(z_1, z_2, \dots, z_n)] - [(x_1, x_2, \dots, x_n)(y_1, y_2, \dots, y_n)(z_1, z_2, \dots, z_n)]) \\ &= \omega_i([x_1(y_1 + c_1)z_1] - [x_1 y_1 z_1], [x_2(y_2 + c_2)z_2] - [x_2 y_2 z_2], \dots, [x_n(y_n + c_n)z_n] - [x_n y_n z_n]) \\ &= max\{\omega_1([x_1(y_1 + c_1)z_1] - [x_1 y_1 z_1]), \omega_2([x_2(y_2 + c_2)z_2] - [x_2 y_2 z_2]), \dots, \omega_n([x_n(y_n + c_n)z_n] - [x_n y_n z_n])\} \\ &\leq max\{\omega_1(c_1), \omega_2(c_2), \dots, \omega_n(c_n)\} \\ &= (\omega_1 \times \omega_2 \times, \dots, \times \omega_n)(c_1, c_2, \dots, c_n) \\ &= \omega_i(c)\end{aligned}$$

Hence, The direct product of cubic lateral ideals of ternary

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near-rings is also a cubic lateral ideal of ternary near-ring.

Theorem 3.10. Let H be a non-empty subset of R. Then H is a lateral ideal of R if and only if the characteristic cubic set $\chi_H =< \bar{\mu}_{\chi_H}, \omega_{\chi_H} >$ of H in R is a cubic lateral ideal of R.

Proof: Assume that H is a lateral ideal of R. Let $x, y \in R$. Suppose that $\bar{\mu}_{\chi_H}(x-y) < min\{\bar{\mu}_{\chi_H}(x), \bar{\mu}_{\chi_H}(y)\}$ and $\omega_{\chi_H}(x-y) > max\{\omega_{\chi_H}(x), \omega_{\chi_H}(y)\}$.
It follows that, $\bar{\mu}_{\chi_H}(x-y) = \bar{0}$, $min\{\bar{\mu}_{\chi_H}(x), \bar{\mu}_{\chi_H}(y)\} = \bar{1}$ and $\omega_{\chi_H}(x-y) = 1, max\{\omega_{\chi_H}(x), \omega_{\chi_H}(y)\} = 0$.
This implies that $x, y \in H$ but $x - y \notin H$ a contradiction. So,
$\bar{\mu}_{\chi_H}(x-y) \geq min\{\bar{\mu}_{\chi_H}(x), \bar{\mu}_{\chi_H}(y)\}$ and $\omega_{\chi_H}(x-y) \leq max\{\omega_{\chi_H}(x), \omega_{\chi_H}(y)\}$

Assume that $\bar{\mu}_{\chi_H}(y+x-y) < \bar{\mu}_{\chi_H}(x)$ and $\omega_{\chi_H}(y+x-y) > \omega_{\chi_H}(x)$.
It follows that $\bar{\mu}_{\chi_H}(y+x-y) = \bar{0}$, $\bar{\mu}_{\chi_H}(x) = \bar{1}$ and $\omega_{\chi_H}(y+x-y) = 1$,
$\omega_{\chi_H}(x) = 0$. So, $x \in H$ but $y + x - y \notin H$ a contradiction to H. Thus $\bar{\mu}(y+x-y) \geq \bar{\mu}(x)$ and $\omega(y+x-y) \leq \omega(x)$
suppose consider $\bar{\mu}([x(y+c)z] - [xyz]) < \bar{\mu}(c)$ and $\omega([x(y+c)z] - [xyz]) > \omega(c)$
this implies that$\bar{\mu}_{\chi_H}(c) = \bar{1}$, $\bar{\mu}_{\chi_H}([x(y+c)z] - [xyz]) = \bar{0}$ and $\omega_{\chi_H}([x(y+c)z] - [xyz]=1$,
$\omega_{\chi_H}(c) = 0$. So, $c \in H$ but $[x(y+c)z] - [xyz] \notin H$ a contradiction. Hence $\bar{\mu}([x(y+c)z] - [xyz]) \geq \bar{\mu}(c)$ and $\omega([x(y+c)z] - [xyz]) \leq \omega(c)$

Therefore $\chi_H =< \bar{\mu}_{\chi_H}, \omega_{\chi_H} >$ is a cubic lateral ideal of R. Conversely, assume that $\chi_H =< \bar{\mu}_{\chi_H}, \omega_{\chi_H} >$ is a cubic lateral-ideal of R, for any subset H of R. Let $x, y \in H$ then $\bar{\mu}_{\chi_H}(x) = \bar{\mu}_{\chi_H}(y) = \bar{1}$ and $\omega_{\chi_H}(x) = \omega_{\chi_H}(y) = 0$, since χ_H is a cubic lateral ideal of R.
$\bar{\mu}_{\chi_H}(x-y) \geq min\{\bar{\mu}_{\chi_H}(x), \bar{\mu}_{\chi_H}(y)\} = \bar{1}$ and $\omega_{\chi_H}(x-y) \leq max\{\omega_{\chi_H}(x), \omega_{\chi_H}(y)\} = 0$.
This implies that $x - y \in H$.
Let $x \in H$ and $y \in R$ be such that $\bar{\mu}_{\chi_H}(x) = \bar{1}$ and $\omega_{\chi_H}(x) = 0$
$\bar{\mu}_{\chi_H}(y+x-y) \geq \bar{\mu}_{\chi_H}(x) = \bar{1}$ and $\omega_{\chi_H}(y+x-y) \leq \omega_{\chi_H}(x) = 0$
Thus$y + x - y \in H$Let $x, y, z \in R$ and $c \in H$ then $\bar{\mu}([x(y+c)z] - [xyz]) \geq \bar{\mu}(c) = \bar{1}$ and $\omega([x(y+c)z] - [xyz]) \leq \omega(c) = 0$ which implies that $[x(y+c)z] - [xyz] \in H$.
Hence H is a lateral ideal of R.

IV. HOMOMORPHISM AND ANTI HOMOMORPHISM OF CUBIC LATERAL IDEALS IN TERNARY NEAR-RINGS

In this section we characterize the properties of homomorphism and anti-homomorphism in cubic lateral ideals between ternary near-rings.

Definition 4.1. [15] Let R and S be any two ternary near-rings. Then a mapping $\theta: R \to S$ is called a ternary near-ring homomorphism if $\theta(x+y) = \theta(x) + \theta(y)$ and $\theta([xyz]) = [\theta(x)\theta(y)\theta(z)]$ for all $x, y, z \in R$.

Definition 4.2. [4] Let R and S be any two ternary near-rings. Then a mapping $\theta: R \to S$ is called a ternary near-ring anti homomorphism if $\theta(x+y) = \theta(y) + \theta(x)$ and $\theta([xyz]) = [\theta(z)\theta(y)\theta(x)]$ for all $x, y, z \in R$.

Definition 4.3. Let f be a mapping from a set R to R_1. Let $\mathcal{A}_1 =< \bar{\mu}_1, \omega_1 >$ be a cubic set of R and $\mathcal{A}_2 =< \bar{\mu}_2, \omega_2 >$ be a cubic set of R_1. Then the pre-image $f^{-1}(\mathcal{A}_2) =< f^{-1}(\bar{\mu}_2), f^{-1}(\omega_2) >$ is a cubic set of R defined by
$$f^{-1}(\mathcal{A}_2)(x) =< f^{-1}(\bar{\mu}_2)(x), f^{-1}(\omega_2)(x) > \\ =< \bar{\mu}_2(f(x)), \omega_2(f(x)) >$$
The image $f(\mathcal{A}_1) =< f(\bar{\mu}_1), f(\omega_1) >$ is a cubic set of R_1 defined by $f(\mathcal{A}_1)(x) =< f(\bar{\mu}_1)(x), f(\omega_1)(y) >$ where
$$f(\bar{\mu}_1)(x) = \begin{cases} \sup_{y \in f^{-1}(x)} \bar{\mu}(y) & if\ f^{-1}(x) \neq \emptyset \\ [0,0] & otherwise \end{cases}$$
$$f(\omega_1)(x) = \begin{cases} \inf_{y \in f^{-1}(x)} \lambda(y) & if\ f^{-1}(x) \neq \emptyset \\ 1 & otherwise \end{cases}$$

Theorem 4.4. Let $f: R \to R_1$ be a homomorphism between ternary near-rings R and R_1. If $\mathcal{A} = < \bar{\mu}, \omega >$ is a cubic lateral ideal of R_1 then $f^{-1}(\mathcal{A}) =< f^{-1}(\bar{\mu}), f^{-1}(\omega) >$ is a cubic lateral ideal of R.

Proof: Let $\mathcal{A} = < \bar{\mu}, \omega >$ is a cubic lateral ideal of R_1. Let $x, y, z, c \in R$. Then
$$f^{-1}(\bar{\mu})(x-y) = \bar{\mu}(f(x-y)) \\ = \bar{\mu}(f(x) - f(y)) \\ \geq min\{\bar{\mu}(f(x)), \bar{\mu}(f(y))\} \\ = min\{f^{-1}(\bar{\mu}(x)), f^{-1}(\bar{\mu}(y))\}$$
$$f^{-1}(\omega)(x-y) = \omega(f(x-y)) \\ = \omega(f(x) - f(y)) \\ \leq max\{\omega(f(x)), \omega(f(y))\} \\ = max\{f^{-1}(\omega(x)), f^{-1}(\omega(y))\}$$
$$f^{-1}(\bar{\mu})(y+x-y) = \bar{\mu}(f(y+x-y)) \\ = \bar{\mu}(f(y) + f(x) - f(y)) \\ \geq \bar{\mu}(f(x)) \\ = f^{-1}(\bar{\mu}(x))$$
$$f^{-1}(\omega)(y+x-y) = \omega(f(y+x-y)) \\ = \omega(f(y) + f(x) - f(y)) \\ \leq \omega(f(x)) \\ = f^{-1}(\omega(x))$$
$$f^{-1}(\bar{\mu})([x(y+c)z] - [xyz])$$

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$= \bar{\mu}(f([x(y+c)z]-[xyz]))$
$= \bar{\mu}([f(x)(f(y)+f(c))f(z)]-[f(x)f(y)f(z)])$
$\geq \bar{\mu}(f(c))$
$= f^{-1}(\bar{\mu}(c))$
$f^{-1}(\omega)([x(y+c)z]-[xyz])$
$= \omega(f([x(y+c)z]-[xyz]))$
$= \omega([f(x)(f(y)+f(c))f(z)]-[f(x)f(y)f(z)])$
$\leq \omega(f(c))$
$= f^{-1}(\omega(c))$

Hence, $f^{-1}(\mathcal{A}) =< f^{-1}(\bar{\mu}), f^{-1}(\omega) >$ is a cubic lateral ideal of R.

Remark 4.5. We prove the converse of the theorem 4.4. by strengthening the condition on f as follows.

Theorem 4.6. Let $f: R \to R_1$ be an onto homomorphism between ternary near-rings R and R_1. If $\mathcal{A} = < \bar{\mu}, \omega >$ is a cubic subset of R_1 such that $f^{-1}(\mathcal{A}) =< f^{-1}(\bar{\mu}), f^{-1}(\omega) >$ is a cubic lateral ideal of R then $\mathcal{A} = < \bar{\mu}, \omega >$ is a cubic lateral ideal of R_1.

Proof: Let $x, y, z, c \in R_1$. Then $f(a) = x, f(b) = y, f(e) = z, f(d) = c$ for some $a, b, e, d \in R$.

$\bar{\mu}(x-y) = \bar{\mu}(f(a)-f(b))$
$= \bar{\mu}(f(a-b))$
$= f^{-1}(\bar{\mu})(a-b)$
$\geq min\{f^{-1}(\bar{\mu})(a), f^{-1}(\bar{\mu})(b)\}$
$= min\{\bar{\mu}(f(a)), \bar{\mu}(f(b))\}$
$= min\{\bar{\mu}(x), \bar{\mu}(y)\}$
$\omega(x-y) = \omega(f(a)-f(b))$
$= \omega(f(a-b))$
$= f^{-1}(\omega)(a-b)$
$\leq max\{f^{-1}(\omega)(a), f^{-1}(\omega)(b)\}$
$= max\{\omega(f(a)), \omega(f(b))\}$
$= max\{\omega(x), \omega(y)\}$
$\bar{\mu}(y+x-y) = \bar{\mu}(f(b)+f(a)-f(b))$
$= \bar{\mu}(f(b+a-b))$
$= f^{-1}(\bar{\mu})(b+a-b)$
$\geq f^{-1}(\bar{\mu})(a)$
$= \bar{\mu}(f(a))$
$= \bar{\mu}(x)$
$\omega(y+x-y) = \omega(f(b)+f(a)-f(b))$
$= \omega(f(b+a-b))$
$= f^{-1}(\omega)(b+a-b)$
$\leq f^{-1}(\omega)(a)$
$= \omega(f(a))$
$= \omega(x)$
$\bar{\mu}([x(y+c)z]-[xyz])$
$= \bar{\mu}([f(a)(f(b)+f(d)f(e)]-[f(a)f(b)f(e)])$
$= \bar{\mu}(f([a(b+d)e]-[abe]))$
$= f^{-1}(\bar{\mu})([a(b+d)e]-[abe])$
$\geq f^{-1}(\bar{\mu})(d)$
$= \bar{\mu}(f(d))$
$= \bar{\mu}(c)$
$\omega([x(y+c)z]-[xyz])$
$= \omega([f(a)(f(b)+f(d)f(e)]-[f(a)f(b)f(e)])$
$= \omega(f([a(b+d)e]-[abe]))$
$= f^{-1}(\omega)([a(b+d)e]-[abe])$
$\leq f^{-1}(\omega)(d)$
$= \omega(f(d))$
$= \omega(c)$

Hence $\mathcal{A} = < \bar{\mu}, \omega >$ is a cubic lateral ideal of R_1.

Theorem 4.7. Let $f: R \to R_1$ be an onto ternary near-ring homomorphism. If $\mathcal{A} = < \bar{\mu}, \omega >$ is a cubic lateral ideal of R then $f(\mathcal{A}) =< f(\bar{\mu}), f(\omega) >$ is a cubic lateral ideal of R_1.

Proof: Let $\mathcal{A} = < \bar{\mu}, \omega >$ be a cubic lateral ideal of R. Since $f(\bar{\mu})(x') = \sup_{f(x)=x'} \bar{\mu}(x)$ and $f(\omega)(x') = \inf_{f(x)=x'} \omega(x)$ for $x' \in R_1$

So $f(\mathcal{A}) =< f(\bar{\mu}), f(\omega) >$ is non-empty. Let $x', y' \in R_1$. Then we have $\{x \mid x \in f^{-1}(x'-y')\} \supseteq \{x-y \mid x \in f^{-1}(x') \text{ and } y \in f^{-1}(y')\}$ and $\{x \mid x \in f^{-1}(x'y')\} \supseteq \{xy \mid x \in f^{-1}(x') \text{ and } y \in f^{-1}(y')\}$

$f(\bar{\mu})(x'-y') = \sup_{f(p)=x'-y'} \bar{\mu}(p)$
$\geq \sup_{f(x)=x', f(y)=y'} \bar{\mu}(x-y)$
$\geq \sup_{f(x)=x', f(y)=y'} min\{\bar{\mu}(x), \bar{\mu}(y)\}$
$= min\left\{\sup_{f(x)=x'} \bar{\mu}(x), \sup_{f(y)=y'} \bar{\mu}(y)\right\}$
$= min\{f(\bar{\mu})(x'), f(\bar{\mu})(y')\}$
$f(\omega)(x'-y') = \inf_{f(p)=x'-y'} \omega(p)$
$\leq \inf_{f(x)=x', f(y)=y'} \omega(x-y)$
$\leq \inf_{f(x)=x', f(y)=y'} max\{\omega(x), \omega(y)\}$
$= max\left\{\inf_{f(x)=x'} \omega(x), \inf_{f(y)=y'} \omega(y)\right\}$
$= max\{f(\omega)(x'), f(\omega)(y')\}$
$f(\bar{\mu})(y'+x'-y') = \sup_{f(p)=y'+x'-y'} \bar{\mu}(p)$
$\geq \sup_{f(x)=x', f(y)=y'} \bar{\mu}(y+x-y)$
$\geq \sup_{f(x)=x'} \bar{\mu}(x)$
$= f(\bar{\mu})(x')$
$f(\omega)(y'+x'-y') = \inf_{f(p)=y'+x'-y'} \omega(p)$
$\leq \inf_{f(x)=x', f(y)=y'} \omega(y+x-y)$
$\leq \inf_{f(x)=x'} \omega(x)$
$= f(\omega)(x')$
$f(\bar{\mu})([x'(y'+c')z']-[x'y'z'])$
$= \sup_{f(p)=([x'(y'+c')z']-[x'y'z'])} \bar{\mu}(p)$
$\geq \sup_{f(x)=x', f(y)=y', f(z)=z', f(c)=c'} \bar{\mu}(([x'(y'+c')z']-[x'y'z'])$
$\geq \sup_{f(c)=c'} \bar{\mu}(c)$
$= f(\bar{\mu})(c')$
$f(\omega)([x'(y'+c')z']-[x'y'z'])$
$= \inf_{f(p)=([x'(y'+c')z']-[x'y'z'])} \omega(p)$
$\leq \inf_{f(x)=x', f(y)=y', f(z)=z', f(c)=c'} \omega(([x'(y'+c')z']-[x'y'z'])$ $\leq$

$\inf_{f(c)=c'} \omega(c)$
$= f(\omega)(c')$
Therefore $f(\mathcal{A}) =< f(\bar{\mu}), f(\omega) >$ is a cubic lateral ideal of R_1.

Theorem 4.8. Let $f: R \to R_1$ be an anti- homomorphism between ternary near-rings R and R_1. If $\mathcal{A} = < \bar{\mu}, \omega >$ is a cubic lateral ideal of R_1 then $f^{-1}(\mathcal{A}) =< f^{-1}(\bar{\mu}), f^{-1}(\omega) >$ is a cubic lateral ideal of R.

Proof: Follows from theorem 4.4. and hence omitted.

Remark 4.9. We can also state the converse of the theorem 4.8. by strengthening the condition on f as follows.

Theorem 4.10. Let $f: R \to R_1$ be an onto anti-homomorphism between ternary near-rings R and R_1. If $\mathcal{A} = < \bar{\mu}, \omega >$ is a cubic set of R_1 such that $f^{-1}(\mathcal{A}) =< f^{-1}(\bar{\mu}), f^{-1}(\omega) >$ is a cubic lateral ideal of R then $\mathcal{A} = < \bar{\mu}, \omega >$ is a cubic lateral ideal of R_1.

Proof: Follows from theorem 4.6. and hence omitted.

Theorem 4.11. Let $f: R \to R_1$ be an onto anti-homomorphism of ternary near-rings R and R_1. If $\mathcal{A} = < \bar{\mu}, \omega >$ is a cubic lateral ideal of R then $f(\mathcal{A}) =< f(\bar{\mu}), f(\omega) >$ is a cubic lateral ideal of R_1.

Proof: Follows from theorem 4.7. and hence omitted.

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