

# Cubic Lateral Ideals in Ternary Near-Rings

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**Abstract:** In this paper, we introduced the notion of cubic lateral ideals in ternary near-rings and obtain some characterizations of cubic lateral ideals in ternary near-rings. Finally, we investigate some related properties using the concepts of cubic homomorphism and anti-homomorphism between ternary near-rings.

**Keywords:** Ternary near-rings, lateral ideal, cubic lateral ideal, cubic homomorphism.

## I. INTRODUCTION

Zadeh [16,17] in 1965 introduced the notion of fuzzy set. The concept of fuzzy subgroup was first introduced by Rosenfeld [11] in 1971. In 1991, Abou-Zaid [2] investigated the ideal of fuzzy subnear-rings and fuzzy ideals in near-rings. Kim et al [8] applied a few concepts of fuzzy ideals of near-rings. Ternary near-ring is the generalized structure of near-ring. Lehmer [9] in 1932 introduced the notion of ternary algebraic system. Dutta et al [5] introduced the concept of ternary semi ring which is a generalization of the ternary ring introduced by Lister [10]. To discuss these results to near-ring using ternary product Warud Nakkhaseen et al [15] have applied the ideal of ternary semi ring to define left ternary near-ring, ternary subnear-ring and their ideals. Thillaigovindan et al [13] discussed the concept of Interval valued fuzzy ideals of near-rings. Jun et al [7] introduced the concept of cubic sets. This structure encompasses interval-valued fuzzy set and fuzzy set. Also Jun et al [6] introduced the notion of cubic subgroups. Chinnadurai et al [3] introduced the notion of cubic bi-ideals in near-rings.

The purpose of this paper to introduce the notion of cubic lateral ideals in ternary near-rings and concept of homomorphism and anti-homomorphism between ternary near-rings. We Investigate some basic results, properties and examples.

## II. PRELIMINARIES

In this section, we present some definitions that are used in the sequel.

**Definition 2.1** [12] Let  $R$  be a non-empty set and  $[ ]$  be an operation defined from  $R \times R \times R$  to  $R$  called a ternary operation. Then  $(R, [ ])$  is a ternary semigroup if for every  $x, y, z, u, v \in R$ ,  $[[xyz]uv] = [x[yzu]v] = [xy[zuv]]$ .

**Definition 2.2** [12] Let  $A, B, C$  be non-empty subsets of a ternary semi group  $R$ . Then  $[ABC] = \{[abc] \in R : a \in A, b \in B, c \in C\}$ .

**Definition 2.3** [14] Let  $R$  be a non-empty set together with a binary operation  $+$  and ternary operation  $[ ]: R \times R \times$

$R$  to  $R$ . Then  $(R, +, [ ])$  is a right (left, lateral) ternary near-ring if

i)  $(R, +)$  is a group (not necessarily abelian)

ii)  $(R, [ ])$  is a ternary semi group

iii)  $[(a + b)cd] = [acd] + [bcd]$ ,

$[cd(a + b)] = [cda] + [cdb]$ ,  $[c(a + b)d] = [cad] +$

$[cbd]$

for every  $a, b, c, d \in R$ .

Throughout this paper,  $R$  denotes a right ternary near-ring.

**Definition 2.4.** [15] A non-empty subset  $S$  of a ternary near-ring  $R$  is called a ternary subnear-ring of  $R$ , if

i)  $x - y \in S \quad \forall x, y \in S$

ii)  $[SSS] \subseteq S$ .

**Definition 2.5** [14] Let  $R$  be a right ternary near-ring, Let  $(I, +)$  be a normal subgroup of  $(R, +)$ . Then  $I$  is called (i) a right ideal of  $R$  if  $[IRR] \subseteq I$ , (ii) a left ideal of  $R$  if  $[xy(z + c)z] - [xyz] \in I$ , (iii) a lateral ideal of  $R$  if  $[x(y + c)z] - [xyz] \in I$  where  $x, y, z \in R, c \in I$ .  $I$  is an ideal of  $R$  if it is a right, lateral and left ideal of  $R$ .

**Definition 2.6.** [4] Let  $I$  be an lateral ideal of  $R$ . For each  $a + I, b + I$  in the factor group  $R/I$ , we define  $(a + I) + (b + I) = (a + b) + I$  and  $(a + I)(b + I) = (ab) + I$ . Then  $R/I$  is a near-ring which we call the residue class near-ring of  $R$  with respect to  $I$ .

**Definition 2.7.** [1] A mapping  $\mu: X \rightarrow [0,1]$  is called a fuzzy subset of  $X$ .

**Definition 2.8.** [4] A fuzzy subset  $\mu$  of  $R$  is called a fuzzy subternary near-ring of  $R$  if

i)  $\mu(x - y) \geq \min\{\mu(x), \mu(y)\}$

ii)  $\mu([xyz]) \geq \min\{\mu(x), \mu(y), \mu(z)\}$

for all  $x, y, z \in R$ .

**Definition 2.9.** [4] A fuzzy subset  $\mu$  of  $R$  is called a fuzzy ideal of  $R$  if

i)  $\mu(x - y) \geq \min\{\mu(x), \mu(y)\}$

ii)  $\mu(y + x - y) \geq \mu(x)$

iii)  $\mu([xyz]) \geq \mu(x)$

iv)  $\mu([xy(z+c)] - [xyz]) \geq \mu(c)$   
 v)  $\mu([x(y+c)z] - [xyz]) \geq \mu(c)$   
 for every  $x, y, z, c \in R$ . If  $\mu$  is a fuzzy left ideal of  $R$  if it satisfies (i), (ii) and (iv). If  $\mu$  is a fuzzy right ideal of  $R$  if it satisfies (i), (ii) and (iii). If  $\mu$  is a fuzzy lateral ideal of  $R$  if it satisfies (i), (ii) and (v).

**Definition 2.10.** [1] Let  $X$  be a non-empty set. A mapping  $\bar{\mu}: X \rightarrow D[0,1]$  is called interval-valued fuzzy set, where  $D[0,1]$  denote the family of all closed sub intervals of  $[0,1]$  and  $\bar{\mu}(x) = [\mu^-(x), \mu^+(x)]$  for all  $x \in X$ , where  $\mu^-$  and  $\mu^+$  are fuzzy subsets of  $X$  such that  $\mu^-(x) \leq \mu^+(x)$  for all  $x \in X$ .

**Definition 2.11.** [6] Let  $X$  be a non-empty set. A cubic set  $\mathcal{A}$  in  $X$  is a structure  $\mathcal{A} = \{ \langle x, \bar{\mu}_A(x), f_A(x) \rangle : x \in X \}$  which is briefly denoted by  $\mathcal{A} = \langle \bar{\mu}_A, f_A \rangle$ , where  $\bar{\mu}_A = [\mu_A^-, \mu_A^+]$  is an interval-valued fuzzy set (briefly, IVF) in  $X$  and  $f$  is a fuzzy set in  $X$ . In this case, we will use  $\mathcal{A}(x) = \langle \bar{\mu}_A(x), f_A(x) \rangle = \langle [\mu^-(x), \mu^+(x)], f_A(x) \rangle \forall x \in X$ .

**Definition 2.12.** [7] For any non-empty subset  $G$  of a set  $X$ , the characteristic cubic set of  $G$  is defined to be a structure

$$\chi_G(x) = \langle x, \bar{\mu}_{\chi_G}(x), \gamma_{\chi_G}(x) : x \in X \rangle$$

which is briefly denoted by

$$\chi_G(x) = \langle \bar{\mu}_{\chi_G}(x), \gamma_{\chi_G}(x) \rangle$$

where

$$\bar{\mu}_{\chi_G}(x) = \begin{cases} [1,1] & \text{if } x \in G \\ [0,0] & \text{otherwise} \end{cases}$$

and

$$\gamma_{\chi_G}(x) = \begin{cases} 0 & \text{if } x \in G \\ 1 & \text{otherwise} \end{cases}$$

### III. MAIN RESULTS

In this section we define cubic lateral ideals in right ternary near-rings and obtain some characterizations of cubic lateral ideals in right ternary near-rings.

**Definition 3.1.** A cubic set  $\mathcal{A} = \langle \bar{\mu}, \omega \rangle$  of  $R$  is called a cubic subternary near-ring of  $R$  if

- i)  $\bar{\mu}(x - y) \geq \min\{\bar{\mu}(x), \bar{\mu}(y)\}$  and  $\omega(x - y) \leq \max\{\omega(x), \omega(y)\}$
- ii)  $\bar{\mu}([xyz]) \geq \min\{\bar{\mu}(x), \bar{\mu}(y), \bar{\mu}(z)\}$  and  $\omega([xyz]) \leq \max\{\omega(x), \omega(y), \omega(z)\}$  for all  $x, y, z \in R$ .

**Definition 3.2.** A cubic set  $\mathcal{A} = \langle \bar{\mu}, \omega \rangle$  of  $R$  is called a cubic ideal of  $R$  if

- i)  $\bar{\mu}(x - y) \geq \min\{\bar{\mu}(x), \bar{\mu}(y)\}$  and  $\omega(x - y) \leq \max\{\omega(x), \omega(y)\}$
- ii)  $\bar{\mu}(y + x - y) \geq \bar{\mu}(x)$  and  $\omega(y + x - y) \leq \omega(x)$
- iii)  $\bar{\mu}([xyz]) \geq \bar{\mu}(x)$  and  $\omega([xyz]) \leq \omega(x)$
- iv)  $\bar{\mu}([xy(z+c)] - [xyz]) \geq \bar{\mu}(c)$  and  $\omega([xy(z+c)] - [xyz]) \leq \omega(c)$
- v)  $\bar{\mu}([x(y+c)z] - [xyz]) \geq \bar{\mu}(c)$  and  $\omega([x(y+c)z] - [xyz]) \leq \omega(c)$

for every  $x, y, z, c \in R$ . The cubic set  $\mathcal{A} = \langle \bar{\mu}, \omega \rangle$  is a cubic

left ideal of  $R$  if it satisfies (i), (ii) and (iv). The cubic set  $\mathcal{A} = \langle \bar{\mu}, \omega \rangle$  is a cubic right ideal of  $R$  if it satisfies (i), (ii) and (iii). The cubic set  $\mathcal{A} = \langle \bar{\mu}, \omega \rangle$  is a cubic lateral ideal of  $R$  if it satisfies (i), (ii) and (v).

**Example 3.3.** Let  $R = \{a, b, c, d\}$  be a set with two binary operations defined as follows

+	a	b	c	d
a	a	b	c	d
b	b	a	d	c
c	c	d	b	a
d	d	c	a	b

·	a	b	c	d
a	a	a	a	a
b	a	a	a	a
c	a	a	a	c
d	a	a	a	d

Define the ternary product  $[ ]$  of  $R$  by  $[xyz] = (x \cdot y) \cdot z$  for every  $x, y, z \in R$ . Then  $(R, +, [ ])$  is a right ternary near-ring.

Define a cubic set  $\mathcal{A} = \langle \bar{\mu}, \omega \rangle$  in  $R$  by  $\bar{\mu}(a) = [0.8, 0.9]$ ,  $\bar{\mu}(b) = [0.5, 0.6]$ ,  $\bar{\mu}(c) = [0.2, 0.3] = \bar{\mu}(d)$  is an interval-valued fuzzy lateral ideal of  $R$  and  $\omega(a) = 0.2$ ,  $\omega(b) = 0.3$ ,  $\omega(c) = 0.7 = \omega(d)$  is a fuzzy lateral ideal of  $R$ . Thus  $\mathcal{A} = \langle \bar{\mu}, \omega \rangle$  is a cubic lateral ideal of  $R$ .

**Definition 3.4.** Let  $\mathcal{A}_i = \langle \bar{\mu}_i, \omega_i \rangle$  be cubic lateral ideals of near-rings  $R_i$  for  $i = 1, 2, 3, \dots, n$ . Then the cubic direct product of  $\mathcal{A}_i, (i = 1, 2, 3, \dots, n)$  is a function

$$(\bar{\mu}_1 \times \bar{\mu}_2 \times \dots \times \bar{\mu}_n) : (R_1 \times R_2 \times \dots \times R_n) \rightarrow D[0,1]$$

and

$$(\omega_1 \times \omega_2 \times \dots \times \omega_n) : R_1 \times R_2 \times \dots \times R_n \rightarrow [0,1]$$

defined by

$$(\bar{\mu}_1 \times \bar{\mu}_2 \times \dots \times \bar{\mu}_n)(x_1, x_2, \dots, x_n) = \min\{\bar{\mu}_1(x_1), \bar{\mu}_2(x_2), \dots, \bar{\mu}_n(x_n)\}$$

and

$$(\omega_1 \times \omega_2 \times \dots \times \omega_n)(x_1, x_2, \dots, x_n) = \max\{\omega_1(x_1), \omega_2(x_2), \dots, \omega_n(x_n)\}.$$

**Lemma 3.5.** Let  $\mathcal{A} = \langle \bar{\mu}, \omega \rangle$  be a cubic lateral ideal of  $R$ . If  $\mathcal{A}(x) \subset \mathcal{A}(y)$  that is

$\bar{\mu}(x) < \bar{\mu}(y)$  and  $\omega(x) > \omega(y)$  then  $\bar{\mu}(x - y) = \bar{\mu}(x) = \bar{\mu}(y - x)$  and  $\omega(x - y) = \omega(x) = \omega(y - x)$

**Proof:** Let  $\mathcal{A} = \langle \bar{\mu}, \omega \rangle$  be a cubic lateral ideal of  $R$ . Let  $x, y \in R$ . Then

$$\begin{aligned} \bar{\mu}(x - y) &\geq \min\{\bar{\mu}(x), \bar{\mu}(y)\} = \bar{\mu}(x) \\ \bar{\mu}(x) &= \bar{\mu}(x - y + y) \\ &= \bar{\mu}((x - y) - (-y)) \\ &= \bar{\mu}((x - y) - (y)) \\ &\geq \min\{\bar{\mu}(x - y), \bar{\mu}(y)\} \\ &= \bar{\mu}(x - y) \end{aligned}$$

again

$$\begin{aligned} \bar{\mu}(y-x) &\geq \min\{\bar{\mu}(y), \bar{\mu}(x)\} = \bar{\mu}(x) \\ \bar{\mu}(x) &= \bar{\mu}(x-y+y) \\ &= \bar{\mu}(y-(y-x)) \\ &\geq \min\{\bar{\mu}(y), \bar{\mu}(y-x)\} \\ &= \bar{\mu}(y-x). \end{aligned}$$

Similarly we can prove the other result.

**Theorem 3.6.** If  $\mathcal{A} = \langle \bar{\mu}, \omega \rangle$  is a cubic lateral ideal of  $R$ , then the set  $R_{\mathcal{A}} = \{x \in R \mid \mathcal{A}(x) = \mathcal{A}(0)\}$  is a lateral ideal of  $R$ .

**Proof:** Since  $\mathcal{A} = \langle \bar{\mu}, \omega \rangle$  be a cubic lateral ideal of  $R$  and  $x, y \in R$ , then  $\mathcal{A}(x) = \mathcal{A}(0)$  and  $\mathcal{A}(y) = \mathcal{A}(0)$ . Suppose  $x, y, z \in R_{\mathcal{A}}$ .

$$\begin{aligned} \bar{\mu}(x) &= \bar{\mu}(y) = \bar{\mu}(z) = \bar{\mu}(c) = \bar{\mu}(0) \quad \text{and} \\ \omega(x) &= \omega(y) = \omega(z) = \omega(c) = \omega(0) \end{aligned}$$

Since,  $\bar{\mu}$  is an i-v fuzzy lateral ideal of  $R$   $\bar{\mu}(x-y) \geq \min\{\bar{\mu}(x), \bar{\mu}(y)\} = \min\{\bar{\mu}(0), \bar{\mu}(0)\} = \bar{\mu}(0)$  and  $\omega$  is a fuzzy lateral ideal of  $R$   $\omega(x-y) \leq \max\{\omega(x), \omega(y)\} = \max\{\omega(0), \omega(0)\} = \omega(0)$

Thus  $x-y \in R_{\mathcal{A}}$ . For every  $y \in R$  and  $x \in R_{\mathcal{A}}$  we have  $\bar{\mu}(y+x-y) \geq \bar{\mu}(x) = \bar{\mu}(0)$  and  $\omega(y+x-y) \leq \omega(x) = \omega(0)$

Thus  $y+x-y \in R_{\mathcal{A}}$ .

$$\begin{aligned} \text{Let } x, y, z \in R \quad \text{and} \quad c \in R_{\mathcal{A}} \\ \bar{\mu}([x(y+c)z] - [xyz]) &\geq \bar{\mu}(c) = \bar{\mu}(0) \quad \omega([x(y+c)z] - [xyz]) \leq \omega(c) = \omega(0) \end{aligned}$$

Thus  $[x(y+c)z] - [xyz] \in R_{\mathcal{A}}$ .

Therefore,  $R_{\mathcal{A}}$  is a lateral ideal of  $R$ .

**Theorem 3.7.** Let  $I$  be a lateral ideal of  $R$ . If  $\mathcal{A} = \langle \bar{\mu}, \omega \rangle$  is a cubic lateral ideal of  $R$ , then the cubic set  $\mathcal{A} = \langle \bar{\mu}, \omega \rangle$  of  $R/I$  defined by  $\bar{\mu}(a+I) = \sup_{x \in I} \bar{\mu}(a+x)$  and  $\omega(a+I) = \inf_{x \in I} \omega(a+x)$  for all  $x \in I$  is a cubic lateral ideal of the residue class ternary near-ring  $R/I$  of  $R$  with respect to  $I$ .

**Proof:** Let  $a, b \in R$  be such that  $a+I = b+I$ . Then  $b = a+y$  for some  $y \in I$ .

$$\begin{aligned} \bar{\mu}(b+I) &= \sup_{x \in I} \bar{\mu}(b+x) \\ &= \sup_{x \in I} \bar{\mu}(a+y+x) \\ &= \sup_{x+y \in I} \bar{\mu}(a+x) \\ &= \bar{\mu}(a+I) \end{aligned}$$

$$\begin{aligned} \omega(b+I) &= \inf_{x \in I} \omega(b+x) \\ &= \inf_{x \in I} \omega(a+y+x) \\ &= \inf_{x+y \in I} \omega(a+x) \\ &= \omega(a+I) \end{aligned}$$

This means that  $\mathcal{A} = \langle \bar{\mu}, \omega \rangle$  is well defined. Let  $x+I, y+I, z+I, c+I \in R/I$

$$\begin{aligned} \bar{\mu}((x+I) - (y+I)) &= \bar{\mu}((x-y)+I) \\ &= \sup_{i \in I} \bar{\mu}((x-y)+i) \\ &= \sup_{i=p-q \in I} \bar{\mu}((x-y) + (p-q)) \\ &= \sup_{p,q \in I} \bar{\mu}((x+p) - (y+q)) \\ &\geq \sup_{p,q \in I} \min\{\bar{\mu}(x+p), \bar{\mu}(y+q)\} \end{aligned}$$

$$\begin{aligned} &= \min\{\sup_{p \in I} \bar{\mu}(x+p), \sup_{q \in I} \bar{\mu}(y+q)\} \\ &= \min\{\bar{\mu}(x+I), \bar{\mu}(y+I)\} \\ \omega((x+I) - (y+I)) &= \omega((x-y)+I) \\ &= \inf_{i \in I} \omega((x-y)+i) \\ &= \inf_{i=p-q \in I} \omega((x-y) + (p-q)) \\ &= \inf_{p,q \in I} \omega((x+p) - (y+q)) \\ &\leq \inf_{p,q \in I} \max\{\omega(x+p), \omega(y+q)\} \\ &= \max\{\inf_{p \in I} \omega(x+p), \inf_{q \in I} \omega(y+q)\} \\ &= \max\{\omega(x+I), \omega(y+I)\} \end{aligned}$$

$$\begin{aligned} \bar{\mu}((y+I) + (x+I) - (y+I)) &= \bar{\mu}((y+x-y)+I) \\ &= \sup_{i \in I} \bar{\mu}((y+x-y)+i) \\ &= \sup_{i=q+p-q \in I} \bar{\mu}((y+x-y) + (q+p-q)) \\ &= \sup_{p \in I, q \in I} \bar{\mu}((y+q) + (x+p) - (y+q)) \\ &\geq \sup_{p \in I} \bar{\mu}(x+p) \\ &= \bar{\mu}(x+I) \\ \omega((y+I) + (x+I) - (y+I)) &= \omega((y+x-y)+I) \\ &= \inf_{i \in I} \omega((y+x-y)+i) \\ &= \inf_{i=q+p-q \in I} \omega((y+x-y) + (q+p-q)) \\ &= \inf_{p \in I, q \in I} \omega((y+q) + (x+p) - (y+q)) \\ &\leq \inf_{p \in I} \omega(x+p) \\ &= \omega(x+I) \\ \bar{\mu}([(x+I)((y+I) + (c+I))(z+I)] - [(x+I)(y+I)(z+I)]) &= \bar{\mu}([(x(y+c)z] - [xyz]) + I) \\ &= \sup_{i \in I} \bar{\mu}([(x(y+c)z] - [xyz]) + i) \\ &\geq \sup_{i \in I} \bar{\mu}([xcz] + [xiz]) \quad \text{since } xiz \in I \\ &= \sup_{i \in I} \bar{\mu}([x(c+i)z]) \\ &\geq \sup_{i \in I} \bar{\mu}(c+i) \\ &= \bar{\mu}(c+I) \\ \omega([(x+I)((y+I) + (c+I))(z+I)] - [(x+I)(y+I)(z+I)]) &= \omega([(x(y+c)z] - [xyz]) + I) \\ &= \inf_{i \in I} \omega([(x(y+c)z] - [xyz]) + i) \\ &\leq \inf_{i \in I} \omega([xcz] + [xiz]) \quad \text{since } xiz \in I \\ &= \inf_{i \in I} \omega([x(c+i)z]) \\ &\leq \inf_{i \in I} \omega(c+i) \\ &= \omega(c+I) \end{aligned}$$

Hence,  $\mathcal{A} = \langle \bar{\mu}, \omega \rangle$  is a cubic lateral ideal of  $R/I$ .

**Theorem 3.8.** If  $\{\mathcal{A}_i\} = \langle \bar{\mu}_i, \omega_i \mid i \in \lambda \rangle$  be a family of cubic lateral ideal of  $R$ , then the cubic set  $\bigcap_{i \in \lambda} \mathcal{A}_i = \langle \bigcap_{i \in \lambda} \bar{\mu}_i, \bigcup_{i \in \lambda} \omega_i \rangle$  is also a cubic lateral ideal of  $R$ , where  $\lambda$  is any index set.

**Proof:** Let  $\mathcal{A}_i = \langle \bar{\mu}_i, \omega_i \mid i \in \lambda \rangle$  be a family of cubic lateral ideals of  $R$ . Let  $x, y, z, c \in R$  and then  $\bar{\mu} = \bigcap \bar{\mu}_i$ ;  $\omega = \bigcup \omega_i$   
 $\bar{\mu}(x) = \bigcap \bar{\mu}_i(x) = (\inf \bar{\mu}_i)(x)$   
 $\omega(x) = \bigcup \omega_i(x) = (\sup \omega_i)(x)$

$$\begin{aligned}
 \omega(x) &= \cup \omega_i(x) = (\sup \omega_i)(x) &= \max\{\omega_1(x_1 - y_1), \omega_2(x_2 - y_2), \dots, \omega_n(x_n - y_n)\} \\
 &= \sup \omega_i(x) \\
 \bar{\mu}(x - y) &= \inf \bar{\mu}_i(x - y) &\leq \max\{\max\{\omega_1(x_1), \omega_1(y_1)\}, \max\{\omega_2(x_2), \omega_2(y_2)\}, \dots, \max\{\omega_n(x_n), \omega_n(y_n)\}\} \\
 &\geq \inf \{\min\{\bar{\mu}_i(x), \bar{\mu}_i(y)\}\} \\
 &= \min\{\inf \bar{\mu}_i(x), \inf \bar{\mu}_i(y)\} \\
 &= \min\{\cap \bar{\mu}_i(x), \cap \bar{\mu}_i(y)\} &\text{and} \\
 \omega(x - y) &= \sup \omega_i(x - y) &= \max\{\max\{\omega_1(x_1), \omega_2(x_2), \dots, \omega_n(x_n)\}, \max\{\omega_1(y_1), \omega_2(y_2), \dots, \omega_n(y_n)\}\} \\
 &\leq \sup\{\max\{\omega_i(x), \omega_i(y)\}\} \\
 &= \max\{\sup \omega_i(x), \sup \omega_i(y)\} \\
 &= \max\{\cup \omega_i(x), \cup \omega_i(y)\} \\
 \omega(x - y) &= \max\{\omega(x), \omega(y)\} \\
 \bar{\mu}(y + x - y) &= \inf\{\bar{\mu}_i(y + x - y)\} &= \max\{(\omega_1 \times \omega_2 \times \dots \times \omega_n)(x_1, x_2, \dots, x_n), (\omega_1 \times \omega_2 \times \dots \times \omega_n)(y_1, y_2, \dots, y_n)\} \\
 &\geq \inf\{\bar{\mu}_i(x)\} \\
 &= \cap \bar{\mu}_i(x) &\omega_i(x - y) &= \max\{\omega_i(x), \omega_i(y)\} \\
 & &\bar{\mu}_i(y + x - y) &= \bar{\mu}_i((y_1, y_2, \dots, y_n) + (x_1, x_2, \dots, x_n) - (y_1, y_2, \dots, y_n)) \\
 & & &= \bar{\mu}_i(y_1 + x_1 - y_1, y_2 + x_2 - y_2, \dots, y_n + x_n - y_n) \\
 \omega(y + x - y) &= \sup\{\omega_i(y + x - y)\} &= \bar{\mu}(x) &= \min\{\bar{\mu}_1(y_1 + x_1 - y_1), \bar{\mu}_2(y_2 + x_2 - y_2), \dots, \bar{\mu}_n(y_n + x_n - y_n)\} \\
 &\leq \sup\{\omega_i(x)\} \\
 &= \cup \omega_i(x) &&\geq \min\{\bar{\mu}_1(x_1), \bar{\mu}_2(x_2), \dots, \bar{\mu}_n(x_n)\} \\
 & & &= (\bar{\mu}_1 \times \bar{\mu}_2 \times \dots \times \bar{\mu}_n)(x_1, x_2, \dots, x_n) \\
 \bar{\mu}([x(y + c)z] - [xyz]) &= \inf \{\bar{\mu}_i([x(y + c)z] - [xyz])\} &= \omega(x) &= \bar{\mu}_i(x) \\
 &\geq \inf\{\bar{\mu}_i(c)\} \\
 &= \cap \bar{\mu}_i(c) &\omega_i(y + x - y) &= \omega_i((y_1, y_2, \dots, y_n) + (x_1, x_2, \dots, x_n) - (y_1, y_2, \dots, y_n)) \\
 & & &= \omega_i(y_1 + x_1 - y_1, y_2 + x_2 - y_2, \dots, y_n + x_n - y_n) \\
 \omega([x(y + c)z] - [xyz]) &= \sup\{\omega_i([x(y + c)z] - [xyz])\} &= \bar{\mu}(c) &= \max\{\omega_1(y_1 + x_1 - y_1), \omega_2(y_2 + x_2 - y_2), \dots, \omega_n(y_n + x_n - y_n)\} \\
 &\leq \sup\{\omega_i(c)\} \\
 &= \cup \omega_i(c) \\
 &= \omega(c) \\
 \text{Hence, } \prod_{i \in \Lambda} \mathcal{A}_i &= \langle \cap_{i \in \Lambda} \bar{\mu}_i, \cup_{i \in \Lambda} \omega_i \rangle \text{ is a cubic lateral ideal of } R. \\
 \text{Theorem 3.9.} & \text{The direct product of cubic lateral ideals of ternary near-rings is also a cubic lateral ideal of ternary near-ring.} \\
 \text{Proof:} & \text{Let } \mathcal{A}_i = \langle \bar{\mu}_i, \omega_i \rangle \text{ be cubic lateral ideals of ternary near-rings } R_i \text{ for } i = 1, 2, 3, \dots, n. \text{ Let } x = (x_1, x_2, \dots, x_n), y = (y_1, y_2, \dots, y_n), z = (z_1, z_2, \dots, z_n), c = (c_1, c_2, \dots, c_n) \in R_1 \times R_2 \times \dots \times R_n. \\
 \bar{\mu}_i(x - y) &= \bar{\mu}_i((x_1, x_2, \dots, x_n) - (y_1, y_2, \dots, y_n)) \\
 &= \bar{\mu}_i(x_1 - y_1, x_2 - y_2, \dots, x_n - y_n) \\
 &= \min\{\bar{\mu}_1(x_1 - y_1), \bar{\mu}_2(x_2 - y_2), \dots, \bar{\mu}_n(x_n - y_n)\} \\
 &\geq \min\{\min\{\bar{\mu}_1(x_1), \bar{\mu}_1(y_1)\}, \min\{\bar{\mu}_2(x_2), \bar{\mu}_2(y_2)\}, \dots, \min\{\bar{\mu}_n(x_n), \bar{\mu}_n(y_n)\}\} \\
 &= \min\{\min\{\bar{\mu}_1(x_1), \bar{\mu}_2(x_2), \dots, \bar{\mu}_n(x_n)\}, \min\{\bar{\mu}_1(y_1), \bar{\mu}_2(y_2), \dots, \bar{\mu}_n(y_n)\}\} \\
 &= \min\{(\bar{\mu}_1 \times \bar{\mu}_2 \times \dots \times \bar{\mu}_n)(x_1, x_2, \dots, x_n), (\bar{\mu}_1 \times \bar{\mu}_2 \times \dots \times \bar{\mu}_n)(y_1, y_2, \dots, y_n)\} \\
 \omega_i(x - y) &= \omega_i((x_1, x_2, \dots, x_n) - (y_1, y_2, \dots, y_n)) \\
 &= \omega_i(x_1 - y_1, x_2 - y_2, \dots, x_n - y_n) \\
 &= \max\{\omega_1(x_1 - y_1), \omega_2(x_2 - y_2), \dots, \omega_n(x_n - y_n)\} \\
 &\leq \max\{\omega_1(x_1), \omega_1(y_1), \omega_2(x_2), \omega_2(y_2), \dots, \omega_n(x_n), \omega_n(y_n)\} \\
 &= \max\{(\omega_1 \times \omega_2 \times \dots \times \omega_n)(x_1, x_2, \dots, x_n), (\omega_1 \times \omega_2 \times \dots \times \omega_n)(y_1, y_2, \dots, y_n)\} \\
 &= \max\{\omega_1(x_1), \omega_2(x_2), \dots, \omega_n(x_n)\} \\
 &= \omega_i(x) \\
 \bar{\mu}_i([x(y + c)z] - [xyz]) &= \bar{\mu}_i([(x_1, x_2, \dots, x_n)(y_1, y_2, \dots, y_n) + (c_1, c_2, \dots, c_n)(z_1, z_2, \dots, z_n)] - [(x_1, x_2, \dots, x_n)(y_1, y_2, \dots, y_n)] \\
 &\quad (z_1, z_2, \dots, z_n)]) \\
 &= \bar{\mu}_i([x_1(y_1 + c_1)z_1] - [x_1y_1z_1], [x_2(y_2 + c_2)z_2] - [x_2y_2z_2], \dots, [x_n(y_n + c_n)z_n] - [x_ny_nz_n]) \\
 &= \min\{\bar{\mu}_1([x_1(y_1 + c_1)z_1] - [x_1y_1z_1]), \bar{\mu}_2([x_2(y_2 + c_2)z_2] - [x_2y_2z_2]), \dots, \bar{\mu}_n([x_n(y_n + c_n)z_n] - [x_ny_nz_n])\} \\
 &\geq \min\{\bar{\mu}_1(c_1), \bar{\mu}_2(c_2), \dots, \bar{\mu}_n(c_n)\} \\
 &= (\bar{\mu}_1 \times \bar{\mu}_2 \times \dots \times \bar{\mu}_n)(c_1, c_2, \dots, c_n) \\
 &= \bar{\mu}_i(c) \\
 \omega_i([x(y + c)z] - [xyz]) &= \omega_i([(x_1, x_2, \dots, x_n)(y_1, y_2, \dots, y_n) + (c_1, c_2, \dots, c_n)(z_1, z_2, \dots, z_n)] - [(x_1, x_2, \dots, x_n)(y_1, y_2, \dots, y_n)] \\
 &\quad (z_1, z_2, \dots, z_n)]) \\
 &= \omega_i([x_1(y_1 + c_1)z_1] - [x_1y_1z_1], [x_2(y_2 + c_2)z_2] - [x_2y_2z_2], \dots, [x_n(y_n + c_n)z_n] - [x_ny_nz_n]) \\
 &= \max\{\omega_1([x_1(y_1 + c_1)z_1] - [x_1y_1z_1]), \omega_2([x_2(y_2 + c_2)z_2] - [x_2y_2z_2]), \dots, \omega_n([x_n(y_n + c_n)z_n] - [x_ny_nz_n])\} \\
 &\leq \max\{\omega_1(c_1), \omega_2(c_2), \dots, \omega_n(c_n)\} \\
 &= (\omega_1 \times \omega_2 \times \dots \times \omega_n)(c_1, c_2, \dots, c_n) \\
 &= \omega_i(c) \\
 \text{Hence, The direct product of cubic lateral ideals of ternary} &
 \end{aligned}$$

near-rings is also a cubic lateral ideal of ternary near-ring.

**Theorem 3.10.** Let  $H$  be a non-empty subset of  $R$ . Then  $H$  is a lateral ideal of  $R$  if and only if the characteristic cubic set  $\chi_H = \langle \bar{\mu}_{\chi_H}, \omega_{\chi_H} \rangle$  of  $H$  in  $R$  is a cubic lateral ideal of  $R$ .

**Proof:** Assume that  $H$  is a lateral ideal of  $R$ . Let  $x, y \in R$ . Suppose  $\bar{\mu}_{\chi_H}(x - y) < \min\{\bar{\mu}_{\chi_H}(x), \bar{\mu}_{\chi_H}(y)\}$  and  $\omega_{\chi_H}(x - y) > \max\{\omega_{\chi_H}(x), \omega_{\chi_H}(y)\}$ .

It follows that  $\bar{\mu}_{\chi_H}(x - y) = \bar{0}$ ,  $\min\{\bar{\mu}_{\chi_H}(x), \bar{\mu}_{\chi_H}(y)\} = \bar{1}$  and  $\omega_{\chi_H}(x - y) = 1$ ,  $\max\{\omega_{\chi_H}(x), \omega_{\chi_H}(y)\} = 0$ .

This implies that  $x, y \in H$  but  $x - y \notin H$  a contradiction. So,

$\bar{\mu}_{\chi_H}(x - y) \geq \min\{\bar{\mu}_{\chi_H}(x), \bar{\mu}_{\chi_H}(y)\}$  and  $\omega_{\chi_H}(x - y) \leq \max\{\omega_{\chi_H}(x), \omega_{\chi_H}(y)\}$

Assume that  $\bar{\mu}_{\chi_H}(y + x - y) < \bar{\mu}_{\chi_H}(x)$  and  $\omega_{\chi_H}(y + x - y) > \omega_{\chi_H}(x)$ .

It follows that  $\bar{\mu}_{\chi_H}(y + x - y) = \bar{0}$ ,  $\bar{\mu}_{\chi_H}(x) = \bar{1}$  and  $\omega_{\chi_H}(y + x - y) = 1$ ,

$\omega_{\chi_H}(x) = 0$ . So,  $x \in H$  but  $y + x - y \notin H$  a contradiction to  $H$ . Thus

$\bar{\mu}(y + x - y) \geq \bar{\mu}(x)$  and  $\omega(y + x - y) \leq \omega(x)$  suppose

$\bar{\mu}([x(y + c)z] - [xyz]) < \bar{\mu}(c)$  and  $\omega([x(y + c)z] - [xyz]) > \omega(c)$

this implies that  $\bar{\mu}_{\chi_H}(c) = \bar{1}$ ,  $\bar{\mu}_{\chi_H}([x(y + c)z] - [xyz]) = \bar{0}$  and  $\omega_{\chi_H}([x(y + c)z] - [xyz]) = 1$ ,

$\omega_{\chi_H}(c) = 0$ . So,  $c \in H$  but  $[x(y + c)z] - [xyz] \notin H$  a contradiction.

Hence  $\bar{\mu}([x(y + c)z] - [xyz]) \geq \bar{\mu}(c)$  and  $\omega([x(y + c)z] - [xyz]) \leq \omega(c)$

Therefore  $\chi_H = \langle \bar{\mu}_{\chi_H}, \omega_{\chi_H} \rangle$  is a cubic lateral ideal of  $R$ . Conversely, assume that  $\chi_H = \langle \bar{\mu}_{\chi_H}, \omega_{\chi_H} \rangle$  is a cubic lateral-ideal of  $R$ , for any subset  $H$  of  $R$ . Let  $x, y \in H$  then  $\bar{\mu}_{\chi_H}(x) = \bar{\mu}_{\chi_H}(y) = \bar{1}$  and  $\omega_{\chi_H}(x) = \omega_{\chi_H}(y) = 0$ , since  $\chi_H$  is a cubic lateral ideal of  $R$ .

$\bar{\mu}_{\chi_H}(x - y) \geq \min\{\bar{\mu}_{\chi_H}(x), \bar{\mu}_{\chi_H}(y)\} = \bar{1}$  and  $\omega_{\chi_H}(x - y) \leq \max\{\omega_{\chi_H}(x), \omega_{\chi_H}(y)\} = 0$ .

This implies that  $x - y \in H$ . Let  $x \in H$  and  $y \in R$  be such that

$\bar{\mu}_{\chi_H}(x) = \bar{1}$  and  $\omega_{\chi_H}(x) = 0$

$\bar{\mu}_{\chi_H}(y + x - y) \geq \bar{\mu}_{\chi_H}(x) = \bar{1}$  and  $\omega_{\chi_H}(y + x - y) \leq \omega_{\chi_H}(x) = 0$

Thus  $y + x - y \in H$ . Let  $x, y, z \in R$  and  $c \in H$  then  $\bar{\mu}([x(y + c)z] - [xyz]) \geq \bar{\mu}(c) = \bar{1}$  and  $\omega([x(y + c)z] - [xyz]) \leq \omega(c) = 0$  which implies that  $[x(y + c)z] - [xyz] \in H$ .

Hence  $H$  is a lateral ideal of  $R$ .

#### IV. HOMOMORPHISM AND ANTI HOMOMORPHISM OF CUBIC LATERAL IDEALS IN TERNARY NEAR-RINGS

In this section we characterize the properties of homomorphism and anti-homomorphism in cubic lateral ideals between ternary near-rings.

**Definition 4.1.** [15] Let  $R$  and  $S$  be any two ternary near-rings. Then a mapping  $\theta: R \rightarrow S$  is called a ternary near-ring homomorphism if  $\theta(x + y) = \theta(x) + \theta(y)$  and  $\theta([xyz]) = [\theta(x)\theta(y)\theta(z)]$  for all  $x, y, z \in R$ .

**Definition 4.2.** [4] Let  $R$  and  $S$  be any two ternary near-rings. Then a mapping  $\theta: R \rightarrow S$  is called a ternary near-ring anti homomorphism if  $\theta(x + y) = \theta(y) + \theta(x)$  and  $\theta([xyz]) = [\theta(z)\theta(y)\theta(x)]$  for all  $x, y, z \in R$ .

**Definition 4.3.** Let  $f$  be a mapping from a set  $R$  to  $R_1$ . Let  $\mathcal{A}_1 = \langle \bar{\mu}_1, \omega_1 \rangle$  be a cubic set of  $R$  and  $\mathcal{A}_2 = \langle \bar{\mu}_2, \omega_2 \rangle$  be a cubic set of  $R_1$ . Then the pre-image  $f^{-1}(\mathcal{A}_2) = \langle f^{-1}(\bar{\mu}_2), f^{-1}(\omega_2) \rangle$  is a cubic set of  $R$  defined by

$$f^{-1}(\mathcal{A}_2)(x) = \langle f^{-1}(\bar{\mu}_2)(x), f^{-1}(\omega_2)(x) \rangle = \langle \bar{\mu}_2(f(x)), \omega_2(f(x)) \rangle$$

The image  $f(\mathcal{A}_1) = \langle f(\bar{\mu}_1), f(\omega_1) \rangle$  is a cubic set of  $R_1$  defined by

$$f(\mathcal{A}_1)(x) = \langle f(\bar{\mu}_1)(x), f(\omega_1)(y) \rangle \text{ where}$$

$$f(\bar{\mu}_1)(x) = \begin{cases} \sup_{y \in f^{-1}(x)} \bar{\mu}(y) & \text{if } f^{-1}(x) \neq \emptyset \\ [0, 0] & \text{otherwise} \end{cases}$$

$$f(\omega_1)(x) = \begin{cases} \inf_{y \in f^{-1}(x)} \omega(y) & \text{if } f^{-1}(x) \neq \emptyset \\ 1 & \text{otherwise} \end{cases}$$

**Theorem 4.4.** Let  $f: R \rightarrow R_1$  be a homomorphism between ternary near-rings  $R$  and  $R_1$ . If  $\mathcal{A} = \langle \bar{\mu}, \omega \rangle$  is a cubic lateral ideal of  $R_1$  then  $f^{-1}(\mathcal{A}) = \langle f^{-1}(\bar{\mu}), f^{-1}(\omega) \rangle$  is a cubic lateral ideal of  $R$ .

**Proof:** Let  $\mathcal{A} = \langle \bar{\mu}, \omega \rangle$  is a cubic lateral ideal of  $R_1$ . Let  $x, y, z, c \in R$ . Then

$$\begin{aligned} f^{-1}(\bar{\mu})(x - y) &= \bar{\mu}(f(x - y)) \\ &= \bar{\mu}(f(x) - f(y)) \\ &\geq \min\{\bar{\mu}(f(x)), \bar{\mu}(f(y))\} \\ &= \min\{f^{-1}(\bar{\mu}(x)), f^{-1}(\bar{\mu}(y))\} \end{aligned}$$

$$\begin{aligned} f^{-1}(\omega)(x - y) &= \omega(f(x - y)) \\ &= \omega(f(x) - f(y)) \\ &\leq \max\{\omega(f(x)), \omega(f(y))\} \\ &= \max\{f^{-1}(\omega(x)), f^{-1}(\omega(y))\} \end{aligned}$$

$$\begin{aligned} f^{-1}(\bar{\mu})(y + x - y) &= \bar{\mu}(f(y + x - y)) \\ &= \bar{\mu}(f(y) + f(x) - f(y)) \\ &\geq \bar{\mu}(f(x)) \\ &= f^{-1}(\bar{\mu}(x)) \end{aligned}$$

$$\begin{aligned} f^{-1}(\omega)(y + x - y) &= \omega(f(y + x - y)) \\ &= \omega(f(y) + f(x) - f(y)) \\ &\leq \omega(f(x)) \\ &= f^{-1}(\omega(x)) \end{aligned}$$

$$f^{-1}(\bar{\mu})([x(y + c)z] - [xyz]) = \bar{\mu}(f([x(y + c)z] - [xyz]))$$

$$\begin{aligned}
 &= \bar{\mu}(f([x(y+c)z] - [xyz])) \\
 &= \bar{\mu}([f(x)(f(y) + f(c))f(z)] - [f(x)f(y)f(z)]) \\
 &\geq \bar{\mu}(f(c)) \\
 &= f^{-1}(\bar{\mu}(c)) \\
 f^{-1}(\omega)([x(y+c)z] - [xyz]) \\
 &= \omega(f([x(y+c)z] - [xyz])) \\
 &= \omega([f(x)(f(y) + f(c))f(z)] - [f(x)f(y)f(z)]) \\
 &\leq \omega(f(c)) \\
 &= f^{-1}(\omega(c))
 \end{aligned}$$

Hence,  $f^{-1}(\mathcal{A}) = \langle f^{-1}(\bar{\mu}), f^{-1}(\omega) \rangle$  is a cubic lateral ideal

$$\begin{aligned}
 &= \omega(f([a(b+d)e] - [abe])) \\
 &= f^{-1}(\omega)([a(b+d)e] - [abe]) \\
 &\leq f^{-1}(\omega)(d) \\
 &= \omega(f(d)) \\
 &= \omega(c)
 \end{aligned}$$

Hence  $\mathcal{A} = \langle \bar{\mu}, \omega \rangle$  is a cubic lateral ideal of  $R_1$ .

**Theorem 4.7.** Let  $f: R \rightarrow R_1$  be an onto ternary near-ring homomorphism. If  $\mathcal{A} = \langle \bar{\mu}, \omega \rangle$  is a cubic lateral ideal of  $R$  then  $f(\mathcal{A}) = \langle f(\bar{\mu}), f(\omega) \rangle$  is a cubic lateral ideal of  $R_1$ .

**Proof:** Let  $\mathcal{A} = \langle \bar{\mu}, \omega \rangle$  be a cubic lateral ideal of  $R$ . Since  $f(\bar{\mu})(x') = f(x)=x \sup \bar{\mu}(x)$  and

$$f(\omega)(x') = f(x)=x \inf \omega(x) \quad \text{for } x' \in R_1$$

So  $f(\mathcal{A}) = \langle f(\bar{\mu}), f(\omega) \rangle$  is non-empty.

Let  $x', y' \in R_1$ . Then we have

$$\begin{aligned}
 &\{x \mid x \in f^{-1}(x' - y')\} \\
 &\supseteq \{x - y \mid x \in f^{-1}(x') \text{ and } y \in f^{-1}(y')\} \\
 &\text{and } \{x \mid x \in f^{-1}(x' y')\}
 \end{aligned}$$

$$\begin{aligned}
 f(\bar{\mu})(x' - y') &= f(p)=x'-y' \sup \bar{\mu}(p) \\
 &\geq f(x)=x', f(y)=y' \sup \bar{\mu}(x - y) \\
 &\geq f(x)=x', f(y)=y' \sup \min\{\bar{\mu}(x), \bar{\mu}(y)\} \\
 &= \min\{f(x)=x' \sup \bar{\mu}(x), f(y)=y' \sup \bar{\mu}(y)\} \\
 &= \min\{f(\bar{\mu})(x'), f(\bar{\mu})(y')\}
 \end{aligned}$$

$$\begin{aligned}
 f(\omega)(x' - y') &= f(p)=x'-y' \inf \omega(p) \\
 &\leq f(x)=x', f(y)=y' \inf \omega(x - y) \\
 &\leq f(x)=x', f(y)=y' \inf \max\{\omega(x), \omega(y)\} \\
 &= \max\{f(x)=x' \inf \omega(x), f(y)=y' \inf \omega(y)\} \\
 &= \max\{f(\omega)(x'), f(\omega)(y')\}
 \end{aligned}$$

$$\begin{aligned}
 f(\bar{\mu})(y' + x' - y') &= f(p)=y'+x'-y' \sup \bar{\mu}(p) \\
 &\geq f(x)=x', f(y)=y' \sup \bar{\mu}(y + x - y) \\
 &\geq f(x)=x' \sup \bar{\mu}(x) \\
 &= f(\bar{\mu})(x')
 \end{aligned}$$

$$\begin{aligned}
 f(\omega)(y' + x' - y') &= f(p)=y'+x'-y' \inf \omega(p) \\
 &\leq f(x)=x', f(y)=y' \inf \omega(y + x - y) \\
 &\leq f(x)=x' \inf \omega(x) \\
 &= f(\omega)(x')
 \end{aligned}$$

$$\begin{aligned}
 f(\bar{\mu})([x'(y'+c')z'] - [x'y'z']) &= f(p)=[x'(y'+c')z']-[x'y'z'] \sup \bar{\mu}(p) \\
 &\geq f(x)=x', f(y)=y', f(z)=z', f(c)=c' \sup \bar{\mu}([x'(y'+c')z'] - [x'y'z']) \\
 &\geq f(c)=c' \sup \bar{\mu}(c) \\
 &= f(\bar{\mu})(c')
 \end{aligned}$$

$$\begin{aligned}
 f(\omega)([x'(y'+c')z'] - [x'y'z']) &= f(p)=[x'(y'+c')z']-[x'y'z'] \inf \omega(p) \\
 &\leq f(x)=x', f(y)=y', f(z)=z', f(c)=c' \inf \omega([x'(y'+c')z'] - [x'y'z']) \\
 &\leq \omega([x'(y'+c')z'] - [x'y'z']) \leq
 \end{aligned}$$

**Remark 4.5.** We prove the converse of the theorem 4.4. by strengthening the condition on  $f$  as follows.

**Theorem 4.6.** Let  $f: R \rightarrow R_1$  be an onto homomorphism between ternary near-rings  $R$  and  $R_1$ . If  $\mathcal{A} = \langle \bar{\mu}, \omega \rangle$  is a cubic subset of  $R_1$  such that  $f^{-1}(\mathcal{A}) = \langle f^{-1}(\bar{\mu}), f^{-1}(\omega) \rangle$  is a cubic lateral ideal of  $R$  then  $\mathcal{A} = \langle \bar{\mu}, \omega \rangle$  is a cubic lateral ideal of  $R_1$ .

**Proof:** Let  $x, y, z, c \in R_1$ . Then  $f(a) = x, f(b) = y, f(e) = z, f(d) = c$  for some  $a, b, e, d \in R$ .

$$\begin{aligned}
 \bar{\mu}(x - y) &= \bar{\mu}(f(a) - f(b)) \\
 &= \bar{\mu}(f(a - b)) \\
 &= f^{-1}(\bar{\mu})(a - b) \\
 &\geq \min\{f^{-1}(\bar{\mu})(a), f^{-1}(\bar{\mu})(b)\} \\
 &= \min\{\bar{\mu}(f(a)), \bar{\mu}(f(b))\} \\
 &= \min\{\bar{\mu}(x), \bar{\mu}(y)\}
 \end{aligned}$$

$$\begin{aligned}
 \omega(x - y) &= \omega(f(a) - f(b)) \\
 &= \omega(f(a - b)) \\
 &= f^{-1}(\omega)(a - b) \\
 &\leq \max\{f^{-1}(\omega)(a), f^{-1}(\omega)(b)\} \\
 &= \max\{\omega(f(a)), \omega(f(b))\} \\
 &= \max\{\omega(x), \omega(y)\}
 \end{aligned}$$

$$\begin{aligned}
 \bar{\mu}(y + x - y) &= \bar{\mu}(f(b) + f(a) - f(b)) \\
 &= \bar{\mu}(f(b + a - b)) \\
 &= f^{-1}(\bar{\mu})(b + a - b) \\
 &\geq f^{-1}(\bar{\mu})(a) \\
 &= \bar{\mu}(f(a)) \\
 &= \bar{\mu}(x)
 \end{aligned}$$

$$\begin{aligned}
 \omega(y + x - y) &= \omega(f(b) + f(a) - f(b)) \\
 &= \omega(f(b + a - b)) \\
 &= f^{-1}(\omega)(b + a - b) \\
 &\leq f^{-1}(\omega)(a) \\
 &= \omega(f(a)) \\
 &= \omega(x)
 \end{aligned}$$

$$\begin{aligned}
 \bar{\mu}([x(y+c)z] - [xyz]) &= \bar{\mu}([f(a)(f(b) + f(c))f(e)] - [f(a)f(b)f(e)]) \\
 &= \bar{\mu}(f([a(b+d)e] - [abe])) \\
 &= f^{-1}(\bar{\mu})([a(b+d)e] - [abe]) \\
 &\geq f^{-1}(\bar{\mu})(d) \\
 &= \bar{\mu}(f(d)) \\
 &= \bar{\mu}(c)
 \end{aligned}$$

$$\begin{aligned}
 \omega([x(y+c)z] - [xyz]) &= \omega([f(a)(f(b) + f(c))f(e)] - [f(a)f(b)f(e)])
 \end{aligned}$$

$$f(c) \stackrel{\text{inf}}{=} \omega(c) \\ = f(\omega)(c')$$

Therefore  $f(\mathcal{A}) = \langle f(\bar{\mu}), f(\omega) \rangle$  is a cubic lateral ideal of  $R_1$ .

**Theorem 4.8.** Let  $f: R \rightarrow R_1$  be an anti-homomorphism between ternary near-rings  $R$  and  $R_1$ . If  $\mathcal{A} = \langle \bar{\mu}, \omega \rangle$  is a cubic lateral ideal of  $R_1$  then  $f^{-1}(\mathcal{A}) = \langle f^{-1}(\bar{\mu}), f^{-1}(\omega) \rangle$  is a cubic lateral ideal of  $R$ .

**Proof:** Follows from theorem 4.4. and hence omitted.

**Remark 4.9.** We can also state the converse of the theorem 4.8. by strengthening the condition on  $f$  as follows.

**Theorem 4.10.** Let  $f: R \rightarrow R_1$  be an onto anti-homomorphism between ternary near-rings  $R$  and  $R_1$ . If  $\mathcal{A} = \langle \bar{\mu}, \omega \rangle$  is a cubic set of  $R_1$  such that  $f^{-1}(\mathcal{A}) = \langle f^{-1}(\bar{\mu}), f^{-1}(\omega) \rangle$  is a cubic lateral ideal of  $R$  then  $\mathcal{A} = \langle \bar{\mu}, \omega \rangle$  is a cubic lateral ideal of  $R_1$ .

**Proof:** Follows from theorem 4.6. and hence omitted.

**Theorem 4.11.** Let  $f: R \rightarrow R_1$  be an onto anti-homomorphism of ternary near-rings  $R$  and  $R_1$ . If  $\mathcal{A} = \langle \bar{\mu}, \omega \rangle$  is a cubic lateral ideal of  $R$  then  $f(\mathcal{A}) = \langle f(\bar{\mu}), f(\omega) \rangle$  is a cubic lateral ideal of  $R_1$ .

**Proof:** Follows from theorem 4.7. and hence omitted.

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