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Higher-Order Single Mode Squeezing For Couple Cavity Optomechanical System

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Abstract: The higher-order single mode squeezing effects of different field modes in couple cavity optomechanical system are investigated using short time dynamics up to the second order Hamiltonian interaction term. The occurrence of amplitude squeezing for both quadratures of different field modes are observed and dependence on the coupling strength, photon hopping interaction term and phase values of the field amplitude are also investigated. Depth of nonclassicalities increases with increasing order.

Keywords: higher-order squeezing, non classical, optomechanics.

I. INTRODUCTION

Non classical effects play an important role in quantum computation and quantum information processing, over last few decades. Squeezing is a non classical phenomenon which can theoretically predicted and also experimentally realizable. In squeezing quantum fluctuations in any field quadrature is less than zero point fluctuation, at the cost of fluctuation for other quadrature. The idea of squeezed states first presented by Kennnard [1] for time evolution of generic Gaussian wave packet of harmonic oscillator and also followed by Husimi[2] and Takahasi[3]. Later, Mandel found squeezed state of second harmonic in nonlinear crystal [4], Hillery defined amplitude-squared squeezing [5]. Hong and Mandel explained the idea of n th-order squeezing as a generalization of the second order squeezing [6, 7]. The existence of squeezed state either predicted or observed in various nonlinear optical processes such as parametric amplification [8], hyper-Raman scattering [9], multi-wave mixing processes [10-12], higher-order harmonic generation [13,14]. The discovery of squeezed state also opens a new window in low noise [15,16] limit and also having different potential applications in optical storage [17], nano-displacement measurement[18], quan-tum cryptography [19], optical communications [20], sensitivity enhancement in gravitational wave detection processes[21,22] and so forth. These facts have motivated us to study nonclassical effects in-terms of single mode squeezing for couple cavity optomechanical system.

Optomechanical systems (OMS) are interested since they lie at the boundary between nano-science and quantum optics and also these systems also provide a good platform to study the quantum effects at macroscopic or mesoscopic scale. In OMS due to interaction between photon and phonon, phonon squeezed states may be observed. Phonon squeezed states have application in reduction of quantum noise in solids [23]. Lower order squeezing effect in couple cavity OMS has been already studied [24] and sometimes it is difficult for detection of weak nonclassicality. Therefore we pay the attention on higher-order squeezing in which depth of nonclassicality is expected to be enhanced. In different optical system higher-order nonclassical studies have been done both experimentally [25] and theoretically [26]. Higher-order nonclassical effects have useful applications in transducer [27], prototype gravitational wave detection [28], microwave sensor [29] etc.

Keeping all these facts in mind we are interested to study higher-order squeezing for couple cavity OMS by solving Hamiltonian of the system analytically. Applying short time dynamics we find out perturbative solutions for different field modes. Using the criteria of higher-order squeezing we have investigated the existence of amplitude-squared squeezing, amplitude-cube squeezing and also n-th order squeezing and discussed the result. Finally, a conclusion is made.

II. GENERAL DEFINITION FOR HIGHER-ORDER SINGLE MODE SQUEEZING

Squeezing is a purely quantum mechanical phenomenon which can't have any classical analogue. To investigate squeezed states we define two quadrature operator X andY, which are real and imaginary parts of the field amplitude. These quadratures satisfy following commutation relation with C operator.

$$[X, Y] = \frac{1}{2}C \qquad (1)$$

For n-th order single mode squeezing the operators are [30]

$$X = \frac{1}{2}\{a^n(t) + a^{\dagger n}(t)\},\ Y = \frac{1}{2i}\{a^n(t) - a^{\dagger n}(t)\}$$
$$C = a^n(t)a^{\dagger n}(t) - a^{\dagger n}(t)a^n(t) \qquad (2)$$

The uncertainty relation associated with the above equation (1) is

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$$\langle(\Delta X(t))^2\rangle\langle(\Delta Y(t))^2\rangle \geq \frac{1}{16} \mid \langle C\rangle \mid^2$$

where $\left\langle(\Delta X(t))^2\right\rangle = \langle X^2(t)\rangle - \langle X(t)\rangle^2$ and also same for $\left\langle(\Delta Y(t))^2\right\rangle$. The system is to be squeezed in X direction if

$$\left\langle(\Delta X(t))^2\right\rangle - \tfrac{1}{4} \mid \langle C\rangle \mid \leq 0 \qquad (3)$$

where equality sign holds for minimum uncertainty states. We define the left hand of equation (3) as S- factor and similar for Y quadrature as Q -factor. For amplitude squared and amplitude squeezing the above equation (3) can be written as

$$S(2) = \left\langle(\Delta X(t))^2\right\rangle - \left\langle N + \tfrac{1}{2}\right\rangle < 0 \qquad (4) \qquad S(3) = \left\langle(\Delta X(t))^2\right\rangle - \langle 9N^2 + 9N + 6\rangle < 0 \quad (5)$$

where N is the number operator for corresponding field mode. Similar definitions are used for Y quadrature in-terms of Q-factor.

III. THE MODEL AND ITS SOLUTION

The model system having two separated optomechanical cavities, each cavity is associated with one fixed mirror and a mechanical resonator. The small amplitude mechanical motion is coupled with cavity field due to radiation pressure. This interaction is achieved by sufficient cooling of mechanical resonator at quantum regime condition. Cavity field of both the cavities are coupled by photon-hopping interaction term (ξ) . The destruction (creation) operators for cavity field modes are a_j $(a_j^\dagger)$ and for mechanical modes are b_j $(b_j^\dagger)$. If l is the dimension of each cavity and x, y be mechanical displacement of the mirror about equilibrium positions then cavity frequencies are $\omega_{c1} = \frac{2\pi c}{l+x}$ and $\omega_{c2} = \frac{2\pi c}{l+y}$, respectively. Without loss of generality, we consider two cavities are identical and x, y be very small w.r.t. dimension of the cavity, so that $\omega_{c1} = \omega_{c2} = \frac{2\pi c}{l} = \omega_c$ i.e. cavity resonance frequency. The frequency of oscillation of the mechanical resonator is characterized by ω_m . The Hamiltonian of the system is $(\hbar = 1)$[31]

$$H = \textstyle\sum_{j=1,2}[\omega_c\, a_j^\dagger a_j + \omega_m b_j^\dagger b_j - g a_j^\dagger a_j\, (b_j^\dagger + b_j)] - \xi(a_1^\dagger a_2 + a_2^\dagger a_1) \qquad (6)$$

The Heisenberg equations of motion corresponding to field modes are

$$\frac{da_j}{dt} = -i\left[\omega_c a_j(t) - g\{a_j(t)b_j^\dagger(t) + a_j(t)b_j(t)\} - \xi\, a_k(t)\right]$$
$$\frac{db_j}{dt} = -i[\omega_m b_j(t) - g a_j^\dagger(t) a_j(t)] \qquad (7)$$

where $j, k = 1,2$ and $j \neq k$.
We assume the perturbative solutions of equations (7) in-terms of time dependent coefficients as

$$\begin{aligned} a_j(t) = {} & f_1 a_j(0) + f_2 a_j(0) b_j^\dagger(0) + f_3 a_j(0) b_j(0) \\ & + f_4 a_k(0) + f_5 a_j(0) b_j^{\dagger 2}(0) + f_6 a_j(0) \\ & + f_7 a_j(0) b_j^\dagger(0) b_j(0) + f_8 a_j(0) b_j^2(0) \\ & + f_9 a_j^\dagger(0) a_j^2(0) + f_{10} a_k(0) b_j^\dagger(0) \\ & + f_{11} a_k(0) b_k^\dagger(0) + f_{12} a_k(0) b_k(0) \\ & + f_{13} a_k(0) b_j(0) + f_{14} a_j(0) \end{aligned}$$

$$b_j(t) = h_1 b_j(0) + h_2 a_j^\dagger(0)\, a_j(0) + h_3 a_j^\dagger(0) a_k(0) + h_4 a_k^\dagger(0) a_j(0) \qquad (8)$$

Where f_i $(i = 1, \dots, 14)$ and h_i $(1, \dots, 4)$ all are function of time (see appendix). To check the validity of the above solutions we use equal time commutation relation i.e. $[a_j(t), a_k^\dagger(t)] = \delta_{jk}$.

In these solutions we terminate the terms up-to quadratic coefficient of the interaction parameters, so that the terms beyond g^2t^2 and ξ^2t^2 are neglected. In our present study we consider single photon strong coupling regime conditions for which coupling strength (g) is larger than cavity decay rate (γ_c) and optical losses are neglected as the system is taken as ideal one.

IV. RESULTS AND DISCUSSIONS

In order to investigate the existence of squeezing, we calculate variances of quadratures by considering that photon and phonon modes are initially in coherent state. Due to interaction with nonlinear medium coherent state changes to superposition of coherent state. So, the product of coherent state is $\mid\xi\rangle = \mid\alpha_1\rangle \otimes \mid\alpha_2\rangle \otimes \mid\beta_1\rangle \otimes \mid\beta_2\rangle$ where $\mid\alpha_1\rangle, \mid\alpha_2\rangle, \mid\beta_1\rangle$ and $\mid\beta_2\rangle$ are the eigenkets of the field operators a_1 , a_2 , b_1 and b_2 respectively. So, $a_1 \mid \xi\rangle = \alpha_1 \mid \xi\rangle$.
Using the criteria (equation 3-5) and solutions of equation (8) we obtain for a_j mode as

$$\begin{pmatrix} S_{a_j}(2) \\ Q_{a_j}(2) \end{pmatrix} = f_3^* f_3\left(1 + \mid\alpha_j\mid^2 + 2\mid\alpha_j{}^2\mid^2\right) + (f_1^* f_6 + c.c. \pm f13 f9 + f13 f2 f3 \alpha j4 + c.c. \quad (9)$$

$$\begin{pmatrix} S_{a_j}(3) \\ Q_{a_j}(3) \end{pmatrix} = 9\left[f_3^* f_3\left(2 + 7\mid\alpha_j\mid^2 + 2\mid\alpha_j{}^3\mid^2\right) + 2 f2{*} f2|\beta j|2 + f1{*} f6 + c.c.(2+3|\alpha j|2+|\alpha j2|2) \pm f15 f9 + f14 f2 f3 \alpha j6 + c.c. \quad (10)$$

Again, for b_j mode we obtain

$$\begin{pmatrix} S_{b_j}(2) \\ Q_{b_j}(2) \end{pmatrix} = 8h_2^* h_2 \mid\alpha_j\mid^2 \mid\beta_j\mid^2 \pm \left\{2h_1^2 h_2^2(1 - |\alpha j|2)|\alpha j|2\beta j2 + c.c. \ (11)\right.$$

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$$\begin{pmatrix} S_{b_j}(n) \\ Q_{b_j}(n) \end{pmatrix} = 2n^2 h_2^* h_2 \mid \alpha_j \mid^{2n} \mid \beta_j \mid^{2n-2} \mp n^2 \{ h_1^{2n-2} h_2^2 \mid \alpha_j|2n-4\beta j2n-2+c.c. \quad (12)$$

with $j = 1,2$.

Equation (9,10) corresponds to amplitude squared and amplitude cube squeezing for a_j mode and equation (11,12) corresponds to amplitude squared squeezing and n-th order single mode squeezing for b_j mode ($n \geq 3$). As the right hand side of equations (9-12) are not simple so these are plotted with dimensionless parameter $\omega_m t$ as shown in figure 1-5.

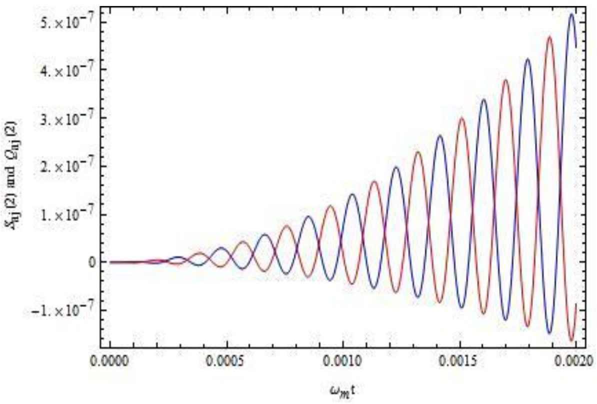

Figure 1: Plot of squeezing parameter $S_{a_j}(2)$ and $Q_{a_j}(2)$ with $\omega_m t$ for a_j mode with n=2 , $\alpha_j = 5$, $\beta_j = 3$, $g = 2\pi \times 0.1$ MHz, $\frac{g}{\xi} = 4$ and $\frac{g}{\omega_m} = 0.8$.

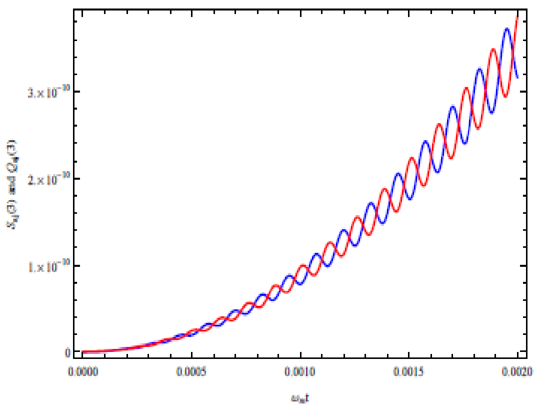

Figure 2: Plot of squeezing parameter $S_{a_j}(3)$ and $Q_{a_j}(3)$ with $\omega_m t$ for a_j mode with n=3 , $\alpha_j = 5$, $\beta_j = 3$, $g = 2\pi \times 0.1$ MHz, $\frac{g}{\xi} = 4$ and $\frac{g}{\omega_m} = 0.8$.

Figure 1 and 2 represents plot of higher-order single mode squeezing for a_j ($j \in 1,2$) mode having order number n=2 and n=3. It is clear that amplitude squared squeezing (n=2) is possible for that mode. The depth of squeezing is small. The amount of squeezing increases with coupling strength g and also with weight factor α_j . The quadrature fluctuations are also manipulated by variation of phase angle φ of the input state that mean if we replace α_j by $\alpha_j e^{i\varphi}$. From figure 3 it is clear that the possibility of getting amplitude cube squeezed state for a_j ($j \in 1,2$) mode is completely ruled out. In explanation it is clear from equation (10) that the contribution of the term $\pm\{(f_1^5 f_9 + f_1^4 f_2 f_3)\alpha_j^6 + c.c.\}$ is very small as compared to the rest the part.

Figure (3-5) represents the possibility of higher order single mode for phonon mode for amplitude squared, n=4 th order and n=5 th order squeezing, respectively. It is clearly observed that depth of squeezing increases with order number very rapidly. It is also observed that the amount of squeezing is also manipulated by coupling strength.

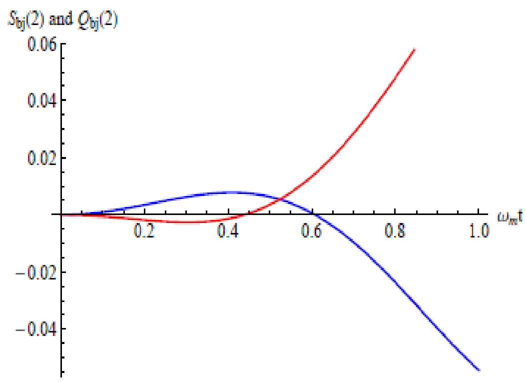

Figure 3: Plot of squeezing parameter $S_{b_j}(2)$ and $Q_{b_j}(2)$ with $\omega_m t$ for b_j mode with n=2 , $\alpha_j = 5$, $\beta_j = 3$, $g = 2\pi \times 0.1$ MHz, $\frac{g}{\xi} = 4$ and $\frac{g}{\omega_m} = 0.8$.

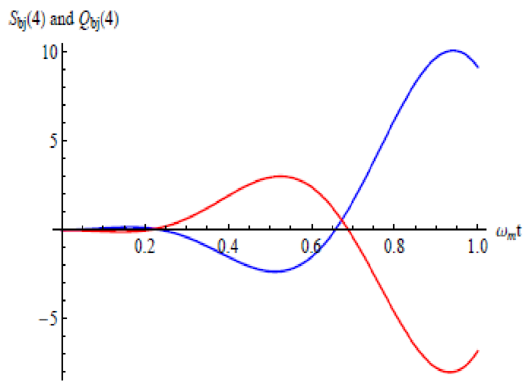

Figure 4: Plot of squeezing parameter $S_{b_j}(4)$ and $Q_{b_j}(4)$ with $\omega_m t$ for b_j mode with n=4 , $\alpha_j = 5$, $\beta_j = 3$, $g = 2\pi \times 0.1$ MHz, $\frac{g}{\xi} = 4$ and $\frac{g}{\omega_m} = 0.8$.

From equation (12) it can be easily state that the term $n^2 \mid \alpha_j \mid^{2n-4}$ plays the amplification factor. This explains that the depth of squeezing how much increases with order number and also with the weight factor α_j. Again it is also seen that as order number increases the time period of quadrature fluctuations are decreases due to energy exchange and more oscillations are produced.

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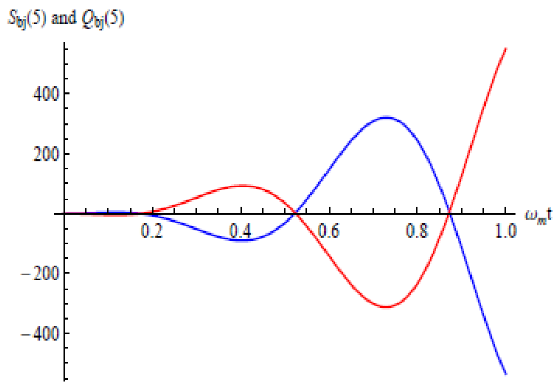

Figure 5: Plot of squeezing parameter $S_{b_j}(5)$ and $Q_{b_j}(5)$ with $\omega_m t$ for b_j mode with n=5 , $\alpha_j = 5$, $\beta_j = 3$, $g = 2\pi \times 0.1$ MHz, $\frac{g}{\xi} = 4$ and $\frac{g}{\omega_m} = 0.8$.

V. CONCLUSION

We explore the possibility of higher-order single mode squeezing for different field modes in terms of amplitude squared squeezing, amplitude cube squeezing and also n-th order squeezing. It is observed that only amplitude squared squeezing is observed for single cavity field mode but amplitude cube squeezing is not observed. Again, for phonon mode squeezing is observed for all order. The effect of squeezing is more pronounced with increasing order number and also with increase in coupling strength. In reference [24] the study shown that lower order single mode squeezing is not possible for couple cavity OMS. But from present study it is clear that single mode squeezing is possible for the system. So the system should be suitable for extracting low noise information regarding higher order single mode squeezing which will be useful for quantum information processing.

REFERENCES

[1] E. H. Kennard, Z.Phys. 44, 326-352, 1927.
[2] K. Husimi, Prog. Theor. Phys. 9, 238-244, 1953.
[3] H. Takahasi, Advances in communication system. Theory and application. Vol. I, 227-310, 1965
[4] L. Mandel, Optics communications, Vol. 42, 6, 437-439, 1982.
[5] M. Hillery, Optics communications, Vol. 62, 2, 135-138, 1987.
[6] C. Hong and L. Mandel, Phy. Rev. Lett. 54, 4, 1985.
[7] C. Hong and L. Mandel, Phy. Rev. A. 32, 2, 974-982, 1985.
[8] L. A. Wu, H. J. Kimble, J. L. Hall and H. Wu, Phy. Rev. Lett. 57, 2520, 1986.
[9] A. Kumar and P. S. Gupta, Journal of optics B: Quantum and semiclassical optics. 8, 5, 1053-1060, 1996.
[10] R. E. Slusher et.al., Phy. Rev. Lett. 55, 22, 1985.
[11] J. Peřina et. al., Optics communications, 49, 4, 1984.
[12] R. Loudon, Optics communications, 49, 1, 1984.
[13] S. T. Gevorkyan, et.al. Optics communications, 134, 440-446, 1984.
[14] G. S. Kanter and P. Kumar, IEEE journal of quantum electronics, 36, 916-922, 8, 2000.
[15] H. J. Kimble and D. F. Walls, J. opt. soc. Am. B, 4, 1450, 1987.
[16] B. E. A. Saleh, M. C. Teich, Phy. Rev. Lett, 58, 2656, 1987.
[17] M. T. L. Hsu. et. al., IEEE journal of quantum electronics, 42, 1001-1007, 2006.
[18] N. Treps et.al., Journal of optics B: Quantum and semiclassical optics. 6, S664-S674, 2004.
[19] J. Kempe, Phy. Rev. A, 60, 910, 1999; C. H. Bennett et. al. , Phy. Rev. Lett, 58, 557, 1992.
[20] H. P. Yuen and J.H. Shapiro, IEEE Transaction on information Theory, 24, 657-668, 1978.
[21] J. Abadie et. al., Nature Physics, 7, 962-965, 2011.
[22] J. Aasi et. al., Nature Photonics, 7, 613-619, 2013.
[23] M. Hillery , Phy. Rev. A, 40, 3147, 1989.
[24] K. Mukherjee and P.C. Jana, Journal of Optics, 016, 0339, 2016.
[25] A. Allevi et. al., Phys. Rev. A. 85, 063835, 2012.
[26] N. B. An, J. Opt. B. Quantum semiclass 4, 222-227, 2002.
[27] K. Stannigel et. al., Phy. Rev. Lett. 105, 22050, 2010.
[28] K. Goda et. al., Nature Physics, 4, 472-476, 2008.
[29] K. Zhang et. al., arxiv: 1401.0070, quant-ph , 2014.
[30] Zhi-ming et.al., Phys. Lett. A. 150, 27, 1990.
[31] J. Q. Liao et. al., arxiv: 1401.1384v1, quant-ph , 2014.

APPENDIX

The time dependent coefficients (in equation 8) are found out from initial boundary condition $f_1(0) = h_1(0) = 1$ and $f_i(0) = 0$ for $i = 2, \ldots, 14$ and $h_i(0) = 0$ for $i = 2, \ldots, 4$. These are as follows:

$$f_1 = e^{-i\omega_c t}$$
$$f_2 = \frac{g}{\omega_m} f_1(F(t) - 1)$$
$$f_3 = \frac{g}{\omega_m} f_1(1 - F^*(t))$$
$$f_4 = i\xi t f_1$$
$$f_5 = \frac{g^2}{2\omega_m^2} f_1(F(t) - 1)^2$$
$$f_6 = \frac{g^2}{\omega_m^2} f_1\{(F^*(t) - 1) + i\omega_m t\}$$
$$f_7 = \frac{g^2}{\omega_m^2} f_1\{F^*(t) + F(t) - 2\}$$
$$f_8 = \frac{g^2}{2\omega_m^2} f_1(F^*(t) - 1)^2$$
$$f_9 = \frac{g^2}{2\omega_m^2} f_1\{F^*(t) - F(t) + 2i\omega_m t\}$$
$$f_{10} = \frac{g\xi}{\omega_m{}^2} f_1\{F(t)(i\omega_m t - 1) + 1\}$$
$$f_{11} = \frac{g\xi}{\omega_m{}^2} f_1\{F(t) - (i\omega_m t - 1)\}$$
$$f_{12} = \frac{g\xi}{\omega_m{}^2} f_1\{F^*(t) + (i\omega_m t - 1)\}$$
$$f_{13} = -\frac{g\xi}{\omega_m{}^2} f_1\{F^*(t)(i\omega_m t + 1) - 1\}$$
$$f_{14} = -\frac{\xi^2 t^2}{2} f_1$$

$$h_1 = F^*(t)$$
$$h_2 = \frac{g}{\omega_m} f_1(1 - h_1)$$
$$h_3 = \frac{g\xi}{\omega_m{}^2} f_1\{h_1 + (i\omega_m t - 1)\}$$
$$h_4 = -h_3$$

where $F(t) = e^{i\omega_m t}$.

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