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A Study on Some Tests of Goodness of Fit For Exponentiality

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Abstract: In many fields like, renewal process, life testing problem, stochastic modeling the assumption of exponentiality is heavily used. In many studies dealing with equipment of failure and repair time ,often these times are assumed to be exponentially distributed. However, considerable efforts have been dedicated to testing for exponentiality. Some of the workers in this fields are Kolmogorov-Smirnov, Von Mises (1931), Bartholomew (1957), Kuiper (1960), Epstein (1954,1960), Watson (1961), Lilliefors (1969), Gail and Gastwirth (1978), Dágostino and Stephen (1976) , Doksum and Yandell (1984) etc which are based on empirical distribution. Among the most recent approaches emphasized are based on entropy estimator ,divergence measures and Kullback-Laibler information's., In this paper we wish to study performance of some of these tests under different alternative hypotheses, viz. under lognormal distribution, Weibull distribution and gamma distributions etc. Results are obtained using Monte Carlo simulation technique and displayed in different tables and graphs. Discussions are made based on simulated results and conclusion is drawn accordingly.

Keywords: Goodness of fit test, Exponential distribution, Lognormal, Weibull and gamma distribution, Monte Carlo technique.

1. INTRODUCTION

The assumption of exponentiality is heavily used in many modeling situations, particularly in life testing and reliability. In many applications, the exponential distribution is widely used for describing a failure mechanism of a system. The distribution is well known as a lifetime model in reliability theory and theoretical justifications for its use as a probability model for failure times of certain component are also well established. Although the distribution plays an important role in modeling failure or life time data as mentioned in Lawless (1982), it is important to assess goodness of fit of the exponential distribution for a data set prior to applying the exponential model in practical applications.

Testing for exponentiality has drawn the attention of many investigators. Standard procedures for checking the validity of the exponential model are the Kolmogorov-Smirnov and Cramer-von Mises which utilize the empirical distribution function(EDF).

Since a series of early works by Epstein and Sobel(1953,1954,1955) and Epstein(1954,1960), various goodness of fit tests based on the empirical distribution function(EDF) have been developed and their power properties were examined through simulations. Lilliefors(1969) suggested a modified Kolmogorov-Smirnov test and tabulated critical values of the test statistic for various sample sizes by Monte Carlo simulations. Van-Soest(1969) studied a modified goodness fit test based on the Cramer-von Mises statistic. Finkelstein and Schafer(1971) and Durbin(1975) improved the previous Kolmogorov-Smirnov statistic and investigated the power of their statistic under several alternative hypotheses through simulations. Gail and Gastwirth (1978) proposed a test for exponentiality based on the Gini's index which is constructed from the area under the lorenze curve. Some standard methods discussed in D'Agostino and Stephens (1986) and Ascher(1990). Recent years, however, have witnessed an increasing interest in using alternative methods, beside those directly involving the density and the distribution function of the exponential model in constructing goodness of fit tests for exponentiality.

These approaches include methods based on entropy and the Kullback-Leibler information (an extended concept of Shannon's entropy) and characterizations involving statistical transformations, such as the Laplace and the Fourier transform. Recently, goodness-of-fit test for exponentiality based on Kullback-Leibler information has been developed by Ebrahimi and Habibullah (1992) and based on Shannon's entropy has been developed by Grzegorzewski and Wieczorkowski (1999). They have used Vasicek's (1976),Van Es'(1992) and Correa's entropy estimators for their study.

The suggested tests have a drawback that the distribution theory related to the sample entropy is difficult, but the powers of the tests estimated by simulations have shown better than those of goodness –of-fit tests based on the empirical distribution. Choi,Kim and Song(2004) deal with testing goodness of fit of exponential distribution based on Kullback-Leibler information which is an extended concept of Shannon's entropy.

The test employ the entropy estimators and the window sizes which must be fixed to compute test statistics for a given sample size. The optimal windows sizes for various sample sizes and the corresponding critical values of each test statistic are determined by means of simulations.

2. TEST PROCEDURES

Let $X_1, X_2, \ldots, X_n$ be a non negative random sample of size n with finite probability density function g(x; .). Let F(x, λ) denote an exponential distribution with a probability density function

$$f(x;\lambda) = \lambda \exp(-\lambda x), \quad \lambda > 0, x \geq 0,$$

and distribution function

$$F(x) = 1 - e^{-\lambda x}$$

where $\lambda = 1/\mu$ is an unknown parameter.

We want to test that given sample of size n come from $F(x) = 1 - e^{-\lambda x}$ against specific general alternatives.

2.1 Kolmogorov Type Statistics

To test the null hypothesis H_0 of exponentiality Kolmogorov type statistic may be used and for this follow the step given below:-

(a) Assume the X_i , i =1,2,…,n are ascending order .

(b) Calculate $\bar{X}$, the mean of the sample and the value $Y_i = X_i / \bar{X}$, i = 1,2,…,n

(c) Calculate $Z_i = 1 - \exp(-Y_i)$, i = 1,2,…,n

2.1.1 The Kolmogorov Statistic D:

(1) Calculate $D^+ = \max(i/n - Z_i)$, $D^- = (Z_i - (i-1)/n)$ and $D = \max(D^+, D^-)$

(2) Modification Calculate

$$D^* = (D - 0.2/n)(\sqrt{n} + 0.26 + 0.5/\sqrt{n}) \quad \ldots \quad (1)$$

(3) Test of H_0, Compare D* with upper tail percentage point of the table value. If D* exceeds a given value reject H_0 at the corresponding significance level.

2.1.2 The Crammer-von Mises Statistic W^2:

(1) Calculate $W^2 = \sum_{i=1}^{n} (Z_i (2i-1)/2n)^2 + 1/(12n)$

(2) Modification calculate

$$W^* = W^2 (1 + 0.16/n) \quad \ldots \quad (2)$$

(3) Test of H_0: Compare W* with its upper tail percentage points of table value at corresponding level of significance.

2.1.3 The Kuiper Statistic V:

(1) Calculate D+, D- as in Kolmogorov statistic and $V = D^+ + D^-$.

(2) Modification Calculate

$$V^* = (V - 0.2/n)(\sqrt{n} + 0.24 + 0.35/\sqrt{n}) \ldots \quad (3)$$

(3) Test of H_0: Compare V* with its upper tail percentage points of table value.

2.1.4 The Watson Statistic U^2:

(1) Calculate W^2 as in Crammer von Mises statistic and then

$$U^2 = W^2 - n(\bar{Z} - \frac{1}{2})^2 , \quad \bar{Z} = \sum_{i=1}^{n} Z_i$$

(2) Modification: Calculate

$$U^* = U^2 (1 + 0.16/n) \ \ldots \quad (4)$$

(3) Test of H_0: Compare U* with its upper tail percentage points of table value.

2.2. Test based on Entropy

The entropy of a random variable was introduced by Shannon(1948) as a measure of information and uncertainty. Now the concept of entropy is one of the fundamental notions of information theory, communication, pattern recognition, statistical physics and stochastic dynamics. In the domain of statistics Shannon's entropy can be used as a descriptive parameter, namely, as a measure of dispersion. Some authors applied Shannon's entropy in the construction of goodness of fit tests. Goodness of fit test for exponentiality can also be constructed by entropy based estimator. It appears that the entropy-based goodness of fit test seems to be a competitive tool for testing exponentiality.

Test construction :

For conditions random variable X with density function f Shannon's entropy is defined as

$$H(x) \equiv H(f) = -\int_{-\infty}^{\infty} f(x) \,\text{In}\, f(x) dx \quad \ldots \quad (5)$$

It is known that if X is a random variable with P(X>0) = 1 and its mean E(X) = λ is given then

$$H(f) \leq 1 + \text{In}\,\lambda \quad \ldots \quad (6)$$

and among all random variable with densities concentrated on (0,+∞) the exponential distribution

$$f_\lambda(x) = \begin{cases} (1/\lambda)\exp(-x/\lambda), \text{if } .x > 0 \\ 0, .otherwise \end{cases} \quad \ldots \quad (7)$$

Maximizes H(f) to $H(f_\lambda) = 1 + \ln \lambda$

After simple transformation we get a following equation

$$\frac{\exp(H(f_\lambda))}{\lambda} = e \quad \ldots \quad (8)$$

This property is used in construction of the test of exponentiality .

Let $X_1, X_2, \ldots, X_n$ denote a sample from the positive continuous distribution with density function f and finite mean. Consider a hypothesis testing problem

$$H_0 : f \in F_{exp} \quad \ldots \quad (9)$$

Where F_{exp} denote a family of exponential distributions with densities (7) against the alternative hypothesis

$$H_1 : f \notin F_{exp} \quad \ldots \quad (10)$$

Goodness of fit test for null hypothesis will be based on a statistic

$$T(X_1, X_2, \ldots, X_n) = \frac{\exp(\hat{H}(X_1, X_2, \ldots, X_n))}{\hat{\lambda}(X_1, X_2, \ldots, X_n)} \ldots \quad (11)$$

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Where $\hat{H}(X_1, X_2, ..., X_n)$ denotes an estimator of entropy and $\hat{\lambda}(X_1, X_2, ..., X_n)$ is an estimator of mean. The null hypothesis H_0 will be rejected in favour of H_1 on the significance level α if T $\le C(\alpha)$, where $C(\alpha)$ is the 100α percentile point of the distribution of T under H_0.

Some appropriate estimator of entropy :

(i) Vasicek (1976) estimator: Let $X_{(1)} \le X_{(2)} \le ... \le X_{(n)}$ denote ordered statistics from he sample $X_1, X_2, \ldots, X_n$. The Vasicek's estimator has a following form

$$\hat{H}_{m,n}(X_1, X_2, ..., X_n) = \frac{1}{n}\sum_{i=1}^{n} \ln\left(\frac{n}{2m}(X_{(i+m)} - X_{(i-m)})\right) \quad (12)$$

Where m is a positive integer smaller than n/2 , $X_{(i)} = X_{(1)}$ for i<1 and $X_{(i)} = X_{(n)}$ for i>n. Farther on test statistics (11) above based on Vasicek's entropy estimator will be denoted by $TV_{m,n}$ and a natural estimator of the mean i.e.

$$\hat{\lambda}(X_1, X_2, \ldots, X_n) = \frac{1}{n}\sum_{i=1}^{n} X_i$$

Thus after easy transformations, we get following formulae for this statistic

$$TV_{m,n} = \frac{n^2}{2m}\left(\sum_{i=1}^{n} X_i\right)^{-1}\left[\prod_{i=1}^{n}(X_{(i+m)} - X_{(i-m)})\right]^{\frac{1}{n}} \quad \ldots (13)$$

(ii) Van Es(1992) Estimator: Van Es proposed an estimator of entropy based on spacings and proved under some conditions consistency and asymptotic normality of the estimator. Van Es estimator is given by

$$\hat{H}_{m,n}(X_1, X_2, ..., X_n) = \frac{1}{n-m}\sum_{i=1}^{n-m} \ln\left(\frac{n+1}{m}(X_{(i+m)} - X_{(i)})\right)$$

$$+ \sum_{k=m}^{n} \frac{1}{k} + \log(m) - \log(n+1) \ \ldots \quad (14)$$

The test statistic (11) based on van Es' entropy estimator will be denoted by TEm,n .

(iii) Correa (1995) Estimator: In a paper Correa(1995) suggested a modification of Vasicek's estimator. It produces smaller mean squared error than Vasicek's estimator. Correa's estimator is given by

$$\hat{H}_{m,n}(X_1, X_2, ..., X_n) = -\frac{1}{n}\sum_{i=1}^{n} \ln(b_i) \quad \ldots \quad (15)$$

where

$$b_i = \frac{\sum_{j=i-m}^{i+m}(X_{(i)} - \bar{X}_{(i)})(j-i)}{n.\sum_{j=i-m}^{i+m}(X_{(j)} - \bar{X}_{(i)})^2}$$

$$\bar{X}_{(i)} = \frac{1}{2m+1}\sum_{j=i-m}^{i+m} X_{(j)}$$

Here, m is a positive integer smaller than n/2 , $X_{(i)} = X_{(1)}$ for i<1 and $X_{(i)} = X_{(n)}$ for i>n.

Farther on statistic (11) based on Correa's entropy estimator will be denoted by $TC_{m,n}$.

2.3 Test based on Kullback-Leibler information

Choi, Kim and Song(2004) proposed a goodness of fit test for exponentiality based on Kullback –Leibler information which is an extended concept of Shannon entropy(1948). Construction of the test is as follows:

Let $X_1, X_2, \ldots, X_n$ be a non-negative random sample of size n with finite mean μ drawn from an unknown continuous distribution F(x; .) with a probability density function f(x; .). Let $F_0(x; \lambda)$ denote an exponential distribution with a probability density function

$$f_0(x; \lambda) = \lambda \exp(-\lambda x), \quad \lambda > 0, x \ge 0 \quad \ldots \quad (16)$$

where $\lambda = 1/\mu$ is an unknown parameter. To construct a goodness of fit tests for exponentiality, we consider the Kullback-Leibler information function defined by

$$I(f{:}f_0) = \int_0^{\infty} f(x;.) \, In \frac{f(x;.)}{f_0(x;\lambda)} dx \quad \ldots \quad (17)$$

The function (17) is a measure of the disparity between F with f(x; .) and F_0 with $f_0(x; \lambda)$ .It is also known that $I(f{:}f_0) \ge 0$, and the equality holds if and only if $f(x;.) = f_0(x; \lambda)$. If a sample comes from an exponential distribution, $I(f : f_0)$ should be close to zero value and thus , large values of $I(f : f_0)$ lead us to reject the null hypothesis $H_0 : F(x; .) = F_0(x; \lambda)$ in favor of the alternative hypothesis $Ha : F(x; .) \ne F(x; \lambda)$.

To derive a test statistic by evaluating the information function (17), a density f must be completely specified. However, in many cases, its form is not known and thus , it is necessary to estimate the information function(17) from a sample. Toward this end,

$I(f{:}f_0)$ is written as

$$I(f{:}f_0) = -H(f) - In\lambda + \lambda\int_0^{\infty} xf(x;.)\,dx = -H(f) - In\lambda + 1, \quad (18)$$

where H(f) is Shannon's entropy of distribution F defined by

$$H(f) = -\int_0^{\infty} f(x;.) \ \ In f(x;.)dx \quad \ldots \quad (19)$$

By the result given in (18) an estimator of $I(f ; f_0)$ can be obtained by replacing individual terms of the right side of (18) by their corresponding estimators.

To get the Kullback –Leibler information, the statistic $1/\bar{X}$ and the entropy estimators proposed by Van Es(1992) and Correa (1995) are used as the estimators of λ and H(f), Van Es entropy estimator based on spacings takes the form of

$$E_m = \frac{1}{n-m}\sum_{i=1}^{n-m} In\left\{\frac{n+1}{m} X_{(i+m)} - X_{(i)}\right\} + \sum_{k=m}^{n}\frac{1}{k} + In\left(\frac{m}{n+1}\right) \cdots \quad (20)$$

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where $X_{(1)} \le X_{(2)} \dots \le X_{(n)}$ is order statistics based on a random sample of size n and window size m is a positive integer smaller than n/2 , $X_{(j)} = X_{(1)}$, if j<1 and $X_{(j)} = X_{(n)}$, if j>n. On the other hand, Correa's entropy estimator which is a modification of Vasicek's one is given by

$$C_{mn} = -\frac{1}{n}\sum In\left\{\frac{\sum_{j=i-m}^{i+m}(X_{(j)} - \bar{X}_{(i)})(j-i)}{n\sum_{i=1}^{n}(X_{(j)} - \bar{X}_{(i)})^2}\right\} \quad \dots (21)$$

$$\text{where } \bar{X}_{(i)} = \sum_{j=i-m}^{i+m} X_{(j)}/(2m+1)$$

Applying a normalizing transformation to the estimated information function, in a similar manner of Ebrahimi and Hbibullah(1992) , the following test statistics are obtained:

$$KLE_{mn} = \exp(E_{mn})/\exp(In\,\bar{X}+1) \quad \dots \quad (22)$$

$$KLC_{mn} = \exp(C_{mn})/\exp(In\,\bar{X}+1) \quad \dots \quad (23)$$

Sufficiently small values of KLE_{mn} or KLC_{mn} indicate that a random sample comes from a non-exponential distribution. Thus, we reject Ho at the significance level α and favour H_1 if $KLE_{mn} \le KLE_{mn}(\alpha)$ or $KLC_{mn} \le KLC_{mn}(\alpha)$, where $KLE_{mn}(\alpha)$ and $KLC_{mn}(\alpha)$ are 100 α percentile of the null distributions of KLE_{mn} and KLC_{mn} , respectively.

2.4 Test Based on Lin-Wong divergence Measure

Abbasnejad, Arghami and Tavakoli (2012) introduce a goodness of fit test for exponentiality based on Lin-Wong divergence measure. This method is similar to Vasicek's method for estimating the Shannon entropy. Lin-Wong (1990) divergence distance of two density functions f(x) and g(x) is given by

$$D_{LW}(f,g) = \int_{-\infty}^{\infty} f(x)\log\frac{2f(x)}{f(x)+g(x)}dx \quad \dots (24)$$

Since Lin-Wong information belongs to Csiszer family, we have $D_{LW}(f,g) \ge 0$ and the equality holds if and only if f(x)=g(x). So, it motivates them to use Lin-Wong information as a test statistic for exponentiality.
Lin-Wong information in favor of f(x) against $f_0(x)$ is

$$D_{LW}(f, f_0) = \int f(x)\log\frac{2f(x)}{f(x)+\lambda e^{-\lambda x}}dx \ . \quad \dots \quad (25)$$

Under the null hypothesis $D_{LW}(f, f_0)=0$ and large values of $D_{LW}(f, f_0)$ favor H_1
To estimate $D_{LW}(f, f_0)$ they use two methods.
In the first method, using F(x) = p, similar to Vasicek's (1976) method they express equation (25) as

$$\int_0^1 \log\frac{2\left(\frac{dF^{-1}(p)}{dp}\right)^{-1}}{\left(\frac{dF^{-1}(p)}{dp}\right)^{-1} + \lambda e^{-(\lambda F^{-1}(p))}}dp$$

Now, replacing F by F_n and using difference operator in place of the differential operator, we get an estimator L_V of $D_{LW}(f, f_0)$ as

$$\mathbf{L_V} = -\frac{1}{n}\sum_{i=1}^{n}\log\left(\frac{1}{2} + \frac{n}{4m\bar{X}}(X_{(i+m)} - X_{(i-m)})e^{-\frac{X_{(i)}}{\bar{X}}}\right) \quad \dots (26)$$

Where $X_{(i)} = X_{(1)}$ for i<1 and $X_{(i)} = X_{(n)}$ for i>n.

Here maximum likelihood estimator $1/\bar{X}$ is used instead of λ in equation (25).

Where $m = [\sqrt{n} + .5]$ .Lv is invariant with respect to scale transformation. Test based on Lv is found to be consistent.
For large values of test statistic we reject the null hypothesis H_0 in favour of H_1 .

2.5. Test based on Cummulative Residual Entropy

Due to certain disadvantages of Shannon entropy Rao et. al.(2004) introduced a new measure of information that extends the Shannon entropy to continuous random variables, and called it cumulative residual entropy (CRE).Based on this new measure, Baratpour and Rad(2012) developed a consistent test statistic for testing the hypothesis of exponentiality against some alternatives. The test statistic is defined as:

$$T_n = \frac{\sum_{i=1}^{n-1}\frac{n-i}{n}\left(In\frac{n-i}{n}\right)(X_{(i+1)} - X_{(i)}) + \frac{\sum_{i=1}^{n}X_i^2}{2\sum_{i=1}^{n}X_i}}{\frac{\sum_{i=1}^{n}X_i^2}{2\sum_{i=1}^{n}X_i}} \quad \dots (27)$$

We reject H_0 at the significance level α and favor H_1 if $T_n \ge T_{n,1-\alpha}$, where $T_{n,1-\alpha}$ is $100(1-\alpha)$ percentile of T_n under H_0.

3. POWER STUDY

In this section we have investigated the performance of goodness-of-fit test based on Vasicek, van Es' and Correa's ,Residual entropy estimators and Divergent measure using Monte Carlo simulations against several alternatives. We consider following alternatives:

- Gamma distribution with density function

$$f(x) = \frac{\lambda^\beta}{\Gamma\beta}e^{-\lambda x}x^{\beta-1} \ , \ \beta > 0, x \ge 0.$$

- Weibull distribution with density function

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$$f(x)=\frac{\beta}{\sigma^{\beta}}x^{\beta-1}\exp[-(\frac{x}{\sigma})^{\beta}]\ ,\ 0\le x<\infty$$

- Lognormal distribution with density function

$$f(x)=\frac{1}{\sqrt{2\pi\sigma_x^2}}\exp\{-\frac{1}{2}[(\frac{\ln x-\mu}{\sigma})^2]\}\ ,\ 0\le x<\infty$$

For each alternative 10000 sample of sizes 5,10,15,20 and 25 were generated and the empirical power of tests were recorded in Table 1-6 by taking the proportion of rejections. This procedure yields absolute errors for the estimated powers less than 0.013 with probability greater or equal to 0.99.

Table 1(a): Empirical Power of Tests for Gamma Distribution

Sample Sizes n	Parameter λ	β	Von-Mises α =.01	.05	Watson .01	.05	K.S .01	.05	Kuiper .01	.05
5	1	2	.0140	.1204	.0286	.1224	.0308	.1188	.0228	.1246
		3	.0244	.2330	.0528	.2306	.0530	.2220	.0410	.2202
		4	.0470	.3546	.1040	.3480	.1036	.3352	.0820	.3336
		5	.0690	.4722	.1458	.4594	.1504	.4394	.1112	.4328
10	1	2	.0556	.2364	.0692	.2142	.0654	.1924	.0568	.2024
		3	.1706	.5230	.1914	.4570	.1716	.4108	.1558	.4354
		4	.3416	.7504	.3768	.6932	.3286	.6336	.2962	.6546
		5	.5208	.8894	.5544	.8498	.5024	.7996	.4544	.8068
15	1	2	.1156	.3618	.1218	.3018	.1080	.2814	.1034	.3032
		3	.3794	.7514	.3712	.6742	.3188	.6152	.3114	.6444
		4	.6824	.9378	.6680	.8954	.5916	.8460	.5704	.8688
		5	.8674	.9868	.8548	.9696	.7924	.9486	.7696	.9556
20	1	2	.1874	.4730	.1782	.3990	.1444	.3626	.1432	.3902
		3	.6078	.8942	.5730	.8322	.4950	.7818	.4974	.8034
		4	.8876	.9888	.8586	.9724	.7920	.9506	.7834	.9612
		5	.9764	.9994	.9658	.9966	.9400	.9910	.9256	.9916
25	1	2	.2650	.5870	.2440	.4992	.2074	.4498	.2062	.4902
		3	.7776	.9592	.7278	.9224	.6496	.8774	.6506	.9038
		4	.9668	.9988	.9510	.9946	.9080	.9864	.9040	.9904
		5	.9970	.9998	.9942	.9984	.9836	.9984	.9804	.9994

Table 1(b): Empirical power of Tests for Gamma distribution

Sample size n	Window m	Parameter λ	β	Tv α =.01	.05	KLCmn .01	.05	KLEmn .01	.05	Tn .01	.05
5	2	1	2	.0710	.1730	.0530	.1660	.0430	.1420	.0470	.1612
		1	3	.0910	.3360	.0760	.3070	.0610	.2260	.0892	.2854
		1	4	.1750	.4920	.1390	.4720	.1020	.3370	.1328	.4654
		1	5	.2600	.6050	.2210	.5740	.1570	.4220	.1990	.5106
10	3	1	2	.1158	.3754	.1060	.3430	.0740	.2360	.0900	.2642
		1	3	.3174	.7110	.2730	.6280	.1710	.4570	.2350	.5136
		1	4	.5458	.8938	.4920	.8110	.3380	.6960	.4038	.7014
		1	5	.7344	.9636	.9710	.9200	.4850	.8380	.5548	.8382
15	4	1	2	.2234	.5248	.2200	.4820	.1090	.3050	.1358	.3444
		1	3	.6050	.8790	.5750	.8410	.3090	.6430	.3682	.6578
		1	4	.8580	.9774	.8090	.9570	.5850	.8680	.6050	.8472
		1	5	.9540	.9976	.9200	.9840	.7770	.9520	.7852	.9480
20	4	1	2	.3236	.7088	.2780	.5550	.1620	.3690	.1738	.3902
		1	3	.8526	.9696	.7130	.9100	.4970	.7790	.4764	.7494
		1	4	.9806	.9984	.9460	.9920	.8070	.9570	.7572	.9266

		1	5	.9976 .9998 .9890 .9980 .9310 .9890 .8976 .9840
25	5	1	2	.3736 .7088 .2780 .5550 .1620 .3690 .1946 .4242
		1	3	.8526 .9696 .7180 .9100 .4970 .7790 .5888 .8266
		1	4	.9806 .9984 .9460 .9920 .8070 .9570 .8586 .9670
		1	5	.9976 .9998 .9890 .9980 .9310 .9890 .9648 .9956

Table 2(a): Empirical Power of Tests for Weibull Distribution

Sample Sizes n	Parameter β	Von-Mises α =.01	.05	Watson .01	.05	K.S .01	.05	Kuiper .01	.05
5	2	.0673	.2962	.0756	.2770	.0080	.0536	.0794	.2640
	3	.2220	.6532	.2478	.6136	.1838	.5412	.2480	.5934
	4	.4382	.8702	.4756	.8414	.3524	.7472	.4804	.8224
	5	.6442	.9586	.6742	.9446	.5142	.8640	.6800	.9352
10	2	.2226	.5906	.2524	.5262	.1878	.5048	.2262	.4966
	3	.7518	.9670	.7726	.9438	.6112	.8904	.7248	.9268
	4	.9646	.9982	.9694	.9968	.8686	.9870	.9560	.9954
	5	.9968	1.000	.9972	1.000	.9672	.9982	.9944	1.000
15	2	.4920	.8186	.4714	.7406	.3786	.6998	.4214	.7040
	3	.9704	.9992	.9620	.9968	.8816	.9860	.9414	.9940
	4	.9996	1.000	.9996	1.000	.9906	.9998	.9986	1.000
	5	1.000	1.000	1.000	1.000	.9996	1.000	1.000	1.000
20	2	.7100	.9332	.6658	.8790	.5604	.8404	.6080	.8476
	3	.9975	1.000	.9954	1.000	.9762	.9988	.9938	.9998
	4	1.000	1.000	1.000	1.000	.9996	1.000	1.000	1.000
	5	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
25	2	.8474	.9762	.7978	.9394	.7108	.9158	.7534	.9248
	3	1.000	1.000	1.000	1.000	.9962	1.000	.9996	1.000
	4	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	5	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000

Table 2(b): Empirical power of Tests for Weibull distribution

Sample size n	Window m	Parameter β	Tv α =.01	.05	KLCmn .01	.05	KLEmn .01	.05	Tn .01	.05
5	2	2	.1372	.3856	.1250	.3880	.0880	.2750	.1360	.3560
		3	.3964	.7556	.3580	.7500	.2370	.5840	.3150	.6830
		4	.6618	.9296	.6190	.9260	.4410	.8110	.5310	.8940
		5	.8392	.9846	.8250	.9840	.6410	.8872	.7812	.9245
10	3	2	.3822	.7622	.3600	.6850	.2180	.5260	.3690	.6260
		3	.8898	.9906	.8670	.9740	.7110	.9240	.8560	.9750
		4	.9906	1.000	.9850	1.000	.9420	.9940	.9850	1.000
		5	1.000	1.000	.9998	1.000	.9988	1.000	.9998	1.000
15	4	2	.6814	.9190	.6160	.8880	.3910	.7000	.5830	.7870
		3	.9958	.9998	.9930	.9990	.9290	.9910	.9860	.9990
		4	.9998	1.000	.9990	1.000	.9990	1.000	1.000	1.000
		5	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
20	4	2	.9050	.9888	.8310	.9650	.6650	.8780	.7150	.8620
		3	1.000	1.000	1.000	1.000	.9990	1.000	.9980	1.000
		4	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000

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		5	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
25	5	2	.9050	.9888	.8310	.9650	.6650	.8780	.8272	.9660
		3	1.000	1.000	1.000	1.000	.9990	1.000	1.000	1.000
		4	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
		5	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000

Table 3(a): Empirical Power of Tests for Lognormal Distribution

Sample Sizes	Parameter		Test Statistics							
n	σ	μ	Von-Mises		Watson		K.S		Kuiper	
			α =.01	.05	.01	.05	.01	.05	.01	.05
5	.4	-.4	.0178	.1770	.0426	.1862	.0464	.1718	.0360	.1610
		-.2	.0436	.3656	.1062	.3624	.1092	.3494	.0810	.3490
		-.1	.0728	.4854	.1532	.4804	.1572	.4530	.1170	.4450
		1	.1448	.7038	.2810	.6942	.2922	.6702	.2290	.6300
		2	.1868	.7840	.3554	.7802	.3594	.7548	.2928	.7628
		4	.2972	.9040	.5048	.8992	.5148	.8798	.4256	.8976
10	.4	-.4	.0988	.3564	.1252	.3354	.1218	.3120	.0996	.3180
		-.2	.3794	.7802	.4192	.7334	.3652	.6800	.3478	.7006
		-.1	.5706	.9096	.6142	.8748	.5426	.8290	.5154	.8518
		1	.8582	.9904	.8830	.9836	.8304	.9702	.8196	.9800
		2	.9328	.9978	.9478	.9946	.9146	.9902	.9108	.9954
		4	.9858	1.000	.9914	.9998	.9814	.9994	.9824	.9998
15	.4	-.4	.2038	.5302	.2272	.4912	.2042	.4592	.1822	.4574
		-.2	.7156	.9418	.7146	.9104	.6334	.8710	.6288	.8862
		-.1	.8912	.9920	.8872	.9834	.8272	.9634	.8244	.9742
		1	.9946	1.000	.9948	.9998	.9824	.9994	.9840	.9998
		2	.9990	1.000	.9990	1.000	.9960	1.000	.9974	1.000
		4	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
20	.4	-.4	.3360	.6622	.3436	.6138	.3142	.5708	.2774	.5756
		-.2	.8996	.9890	.8850	.9802	.8184	.9598	.8160	.9724
		-.1	.9840	.9988	.9824	.9978	.9580	.9948	.9600	.9974
		1	1.000	1.000	1.000	1.000	.9982	1.000	.9996	1.000
		2	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
		4	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
25	.4	-.4	.4698	.7724	.4586	.7204	.4236	.6862	.3896	.6984
		-.2	.9734	.9978	.9646	.9944	.9318	.9898	.9344	.9930
		-.1	.9982	1.000	.9968	1.000	.9926	.9998	.9932	.9998
		1	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
		2	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
		4	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000

Table 3(b): Empirical power of Tests for Lognormal distribution

Sample size	Window	Parameter		Test Statistics							
		σ	μ	Tv		KLCmn		KLEmn		Tn	
n	m			α =.01	.05	.01	.05	.01	.05	.01	.05
5	2	.4	-.4	.0764	.2402	.0602	.2334	.0422	.1770	.060	.2290
		.4	-.2	.1880	.5028	.1548	.4736	.1050	.3642	.1620	.4170
		.4	-.1	.2682	.6408	.2202	.5982	.1464	.4598	.2130	.5200
		.4	.1	.4526	.8286	.3798	.7962	.2560	.6492	.3460	.6890
		.4	.2	.5474	.8952	.4616	.8652	.3176	.7314	.4090	.7520
		.4	.4	.7062	.9630	.6218	.9426	.4406	.8492	.5460	.8530
10	3	.4	-.4	.1812	.3756	.1730	.4092	.1026	.3244	.1330	.3690
		.4	-.2	.5764	.8230	.5228	.8190	.3858	.7386	.4190	.6850

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		.4		-.1	.7618	.9348	.7038	.9260	.5558	.8706	.5480	.7940
		.4	.1		.9478	.9940	.9200	.9916	.8298	.9824	.7660	.9350
		.4	.2		.9780	.9978	.9640	.9982	.9020	.9932	.8360	.9660
		.4	.4		.9972	.9998	.9930	.9998	.9766	.9998	.9370	.990
15	4	.4	-.4		.2532	.5164	.2130	.4132	.1752	.4152	.2190	.4550
		.4	-.2		.8012	.9574	.7066	.8886	.6486	.8942	.5720	.8180
		.4		-.1	.9402	.9936	.8732	.9684	.8382	.9772	.7330	.9170
		.4	.1		.9968	.9998	.9856	.9986	.9848	.9998	.9300	.9870
		.4	.2		.9994	1.000	.9974	.9998	.9980	1.000	.9620	.9970
		.4	.4		1.000	1.000	1.000	1.000	1.000	1.000	.9930	.9990
20	4	.4	-.4		.3792	.6044	.3048	.5320	.2776	.5498	.2740	.5310
		.4	-.2		.9438	.9886	.8806	.9738	.8590	.9822	.7110	.9050
		.4		-.1	.9908	.9988	.9762	.9958	.9752	.9988	.8740	.9550
		.4	.1		.9998	1.000	.9994	1.000	.9998	1.000	.9660	.9980
		.4	.2		1.000	1.000	1.000	1.000	1.000	1.000	.9940	.9990
		.4	.4		1.000	1.000	1.000	1.000	1.000	1.000	.9990	1.000
25	5	4	-.4		.4238	.6660	.3492	.5706	.3848	.6644	.3350	.5970
		.4	-.2		.9698	.9938	.9412	.9866	.9634	.9968	.8210	.9510
		.4		-.1	.9958	.9998	.9896	.9986	.9868	1.000	.9300	.9870
		.4	.1		1.000	1.000	1.000	1.000	1.000	1.000	.9930	.9990
		.4	.2		1.000	1.000	1.000	1.000	1.000	1.000	.9970	1.000
		.4	.4		1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000

Fig.1 Empirical Power of the Test under Gamma altenative at 0.05 and n=15

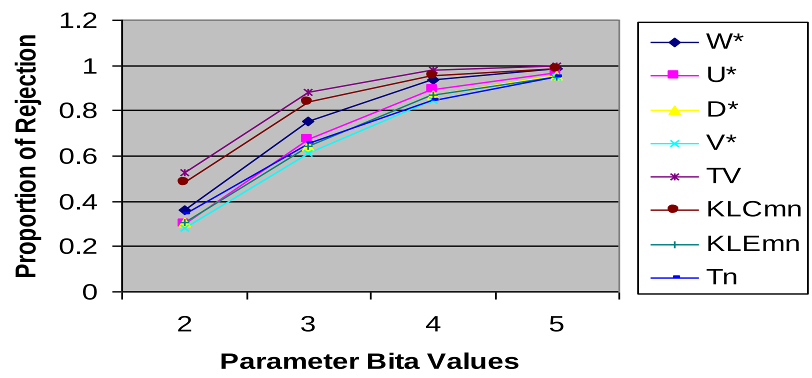
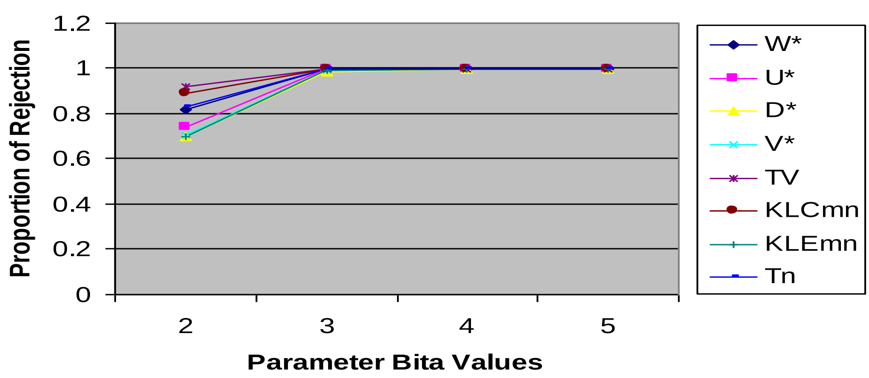
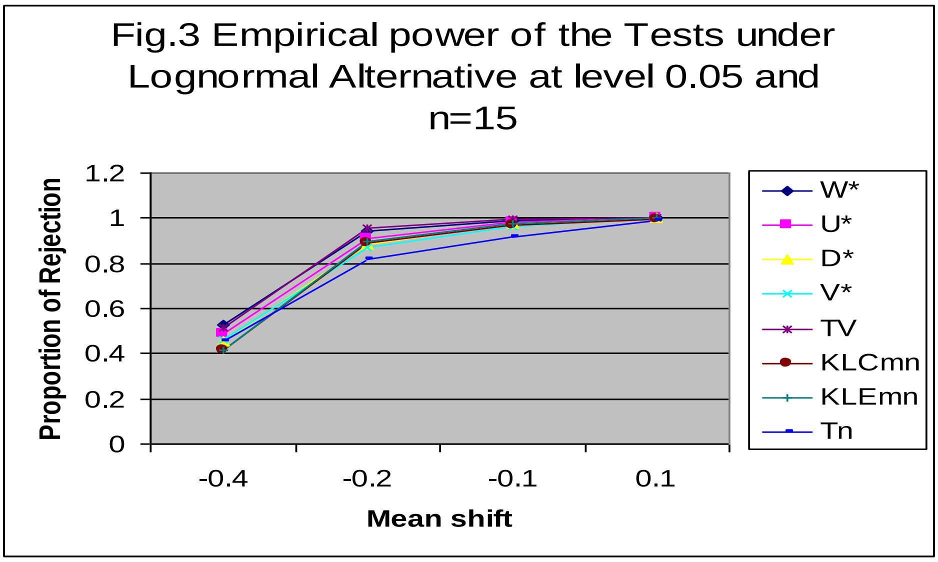


Fig.2 Empirical Power of Test under Weibul alternative at level 0.05 and n=15





4. DISCUSSION ON RESULTS

The power of the tests at the significance level $\alpha = 0.05$ and 0.01 and for selected values of parameters are given in table 1-6. Table 1 shows the empirical powers under gamma alternative for Cramer-von Mises,Watson, Kolmogorov-Smirnov and Kuiper tests based on empirical distribution function. It is seen that power of Cramer von Mises test is slightly more than the other tests. However, as the value of parameter increases empirical power of all the tests become closed to each other. Table 2 shows the empirical power of tests based on entropy estimator under gamma alternative. From the Table 2 it is observe that empirical power of Tv test is more than the other test . However, in presence of large values of parameter all the test comes to close each other.

Table 3 and 4 show the empirical power of tests based on empirical distribution function and based on entropy under Weibul distribution. It is seen that empirical power of Lv test based on divergence measure seems to be more than the other tests based on entropy. Any way empirical power of the KLCmn , based on Correa's entropy is slightly less than the Lv tests. Empirical power of Tn based on cumulative residual entropy seems to be less than all other tests based on entropy. Table 5 and 6 show the empirical power of tests based on empirical function and entropy based estimator under lognormal distribution. It is seen that among the four tests based empirical distribution function Cramer von Mises test is slightly more powerful than the other three tests for small value of parameter .For the large value of parameter empirical power of all the tests are closed to each other. Out of four entropy based tests, Tv test is more powerful than all other test investigated here. It is also observed that the power of all tests against any alternative shows an increasing pattern for the sample size.

5. CONCLUSION

Performance of tests based on appropriate entropy is seems to be better than the other tests. So, one may used such type test for goodness of fit of exponentiality. Only necessary that appropriate entropy estimator should be used to make more efficient.

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