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Exact Solutions of Bianchi Type I Wet Dark Energy in f(R, T) Gravity

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Abstract: Here we have investigated the exact solutions of Bianchi Type I space time in the context of f (R, T) gravity with the matter wet dark energy. The physical nature of this model is also discussed.

Keywords: Bianchi Type I, wet dark matter, f (R, T) gravity.

1. INTRODUCTION

Resent observational data indicates that our universe is accelerating [1-9]. So many researchers are trying to describe late inflation of universe with two different ways some of them by modifying Einstein's general theory of relativity and others by introducing an exotic type of matter viz. a cosmological constant or quintessential type at scalar field. By using cosmological constant we will face some theoretical problems. Therefore some researchers are used various alternatives namely K-essence [10], Tachyon [11], phantom [12] and quintom [13]. Harko et al. [14] proposed a new generalized theory known as f(R,T) gravity. In f(R, T) gravitation theory gravitational Lagrangian involves an arbitrary scalar curvature function R and trace T of the energy momentum tensor T_{ij}. Myrzakulov T. [15] has observed f(R,T) gravity in which he has explicitly presented point like Lagrangians. Adhov [16] has obtained exact solutions at LRS Bianchi type I space time in f(R,T) theory of gravitation. Sharif and Zubair [17] observed the law of thermodynamics in this theory. Haurndjo [18] proved that f(R, T) theory of gravitation allowed transition of matter from dominated stage to an acceleration stage because of this f (R, T) gravity can explain the phase of cosmic explanation of our universe. M.F. Shamir et al. [19] also studied exact solutions of Bianchi type I & V model in f (R, T) gravity and also discussed some physical behaviors of these model.

Mahanta K.L. [20] has studied Locally Rotationally Symmetric Bianchi Type I cosmological models in the f (R, T) theory of gravity with the matter bulk viscous fluid. Bishi B.K. and Mahanta K.L. [21] studied the Bianchi type – V string cosmological model with bulk viscosity in f (R, T) theory of gravity and observed that the cosmic string does not exist in this model. Shamir M.F. [22] investigated the exact solutions of Bianchi type I space time in the context of f (R, T) gravity and obtained two exact solutions by using assumption of constant deceleration parameter and variation law of Hubble parameter. Further he [23] has investigated exact solution of LRS Bianchi Type I universe in this theory and modified field equations are solved by assuming an expansion scalar θ proportional to the shear scalar σ.

Here our aim is to investigate the exact solution at Bianchi type I cosmological model with the matter wet dark energy within the frame work of f (R, T) gravitation theory.

2. F (R, T) THEORY OF GRAVITATION

The f (R, T) theory of gravitation is the generalization and modification of general theory of relativity. For this theory the action is given by.

$$S=\int\sqrt{-g}\left(\frac{1}{16\pi G}f\left(R,T\right)+L_m\right)d^4x \qquad (2.1)$$

where f (R, T) is an arbitrary function of Ricci Scalar R and the trace T of energy momentum tensor T_{ij} while L_m is the usual matter Lagrangian. It is worth mentioning that if we replace f (R,T) with f (R), we get the action for f (R) gravity and replacement of f (R,T) with R leads to the action of general theory of relativity. The energy momentum tensor T_{ij} is defined [24] as

$$T_{ij}=-\frac{2}{\sqrt{-g}}\frac{\delta\left(\sqrt{-g}L_m\right)}{\delta g^{ij}} \qquad (2.2)$$

Here, we assume that the matter Lagrangian is depending on the metric tensor g_{ij} rather than its derivatives. Thus we obtain

$$T_{ij}=L_m g_{ij}-2\frac{\delta L_m}{\delta g^{ij}} \qquad (2.3)$$

The f (R, T) gravity field equations can be obtained by varying the action S in equation (2.1) with respect to the metric tensor g_{ij}

$$f_R(R,T)R_{ij}-\frac{1}{2}f(R,T)g_{ij}-(\nabla_i\nabla_j-g_{ij}\Box)f_R(R,T)=\kappa T_{ij}-f_T(R,T)(T_{ij}+\Theta_{ij}) \qquad (2.4)$$

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Where ∇_i denotes the covariant derivative and

$$\Box \equiv \nabla^i \nabla_i \ , \ f_R(R,T) = \frac{\partial f_R(R,T)}{\partial R},$$

$$f_T(R,T) = \frac{\partial f_R(R,T)}{\partial T},$$

and $$\Theta_{ij} = g^{\alpha\beta}\frac{\delta T_{\alpha\beta}}{\delta g^{ij}}$$

After contraction equation (2.4) given by

$$f_R(R,T)R + 3\Box f_R(R,T) - 2f(R,T) = \kappa T - f_T(R,T)(T+\Theta) \quad (2.5)$$

Where $\Theta = \Theta_i^i$, this is an important equation because it provides a relationship between Ricci scalar R and the trace T of energy momentum tensor.

With the help of matter Lagrangian L_m, we obtain the standard energy momentum tensor for wet dark matter as

$$T_{ij} = (\rho_{wdf} + p_{wdf})v_i v_j - p_{wdf} g_{ij} \quad (2.6)$$

Where $v_i = \sqrt{g_{44}}(1,0,0,0)$ is the four velocity in co-moving co-ordinates and ρ_{wdf} and p_{wdf} denotes energy density and proper pressure of the matter respectively. We can assume the matter Lagrangian as $L_m = -p_{wdf}$ which gives us

$$\Theta_{ij} = -p_{wdf} g_{ij} - 2T_{ij} \quad (2.7)$$

And consequently the field equation (2.4) takes the form.

$$f_R(R,T)R_{ij} - \frac{1}{2}f(R,T)g_{ij} - (\nabla_i\nabla_j - g_{ij}\Box)f_R(R,T) = \kappa T_{ij} + f_T(R,T)(T_{ij} + p_{wdf}g_{ij}) \quad (2.8)$$

Harko et al. have given three classes of these models which are

$$f(R,T) = \begin{cases} R + 2f(T), \\ f_1(R) + f_2(T), \\ f_1(R) + f_2(R)f_3(T). \end{cases} \quad (2.9)$$

Here we will consider the first class of equation (2.9) i.e.

$$f(R,T) = R + 2f(T)$$

For this model the field equation becomes

$$R_{ij} - \frac{1}{2}Rg_{ij} = \kappa T_{ij} + 2f'(T)T_{ij} + \left[f(T) + 2p_{wdf}f'(T)\right]g_{ij} \quad (2.10)$$

Where prime denotes derivative with respect to T.

3. EXACT WET DARK SOLUTION OF BIANCHI TYPE I MODEL

For Bianchi type I cosmological model, the line element is given by

$$ds^2 = dt^2 - A^2(t)dx^2 - B^2(t)dy^2 - C^2(t)dz^2 \quad (3.1)$$

Here, we shall find exact solutions of Bianchi type I space time in $f(R,T)$ theory of gravitation. Now we use natural system of units $(G = C = 1)$ and $f(T) = \mu T$, where μ is an arbitrary constant.

For Bianchi Type I model Ricci Scalar R is defined as

$$R = -2\left[\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\ddot{C}}{C} + \frac{\dot{A}\dot{B}}{AB} + \frac{\dot{B}\dot{C}}{BC} + \frac{\dot{C}\dot{A}}{CA}\right] \quad (3.2)$$

Where

$$\dot{A} = \frac{\partial A}{\partial T}, \quad \ddot{A} = \frac{\partial^2 A}{\partial t^2} \text{ etc}$$

Now, the energy movement tensor for the matter wet dark energy is defined as in equation (2.6)

With the help of equations (2.6), (3.1), (3.2) the field equation (2.10) gives

$$\frac{\dot{A}\dot{B}}{AB} + \frac{\dot{A}\dot{C}}{AC} + \frac{\dot{B}\dot{C}}{BC} = (8\pi + 3\lambda)\rho_{wdf} - p_{wdf}\lambda \quad (3.3)$$

$$\frac{\ddot{B}}{B} + \frac{\ddot{C}}{C} + \frac{\dot{B}\dot{C}}{BC} = -(8\pi + 3\lambda)p_{wdf} + \lambda\rho_{wdf} \quad (3.4)$$

$$\frac{\ddot{C}}{C} + \frac{\ddot{A}}{A} + \frac{\dot{C}\dot{A}}{CA} = -(8\pi + 3\lambda)p_{wdf} + \lambda\rho_{wdf} \quad (3.5)$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\dot{A}\dot{B}}{AB} = -(8\pi + 3\lambda)p_{wdf} + \lambda\rho_{wdf} \quad (3.6)$$

Now, solving equations (3.3) to (3.6) we get

$$\frac{\ddot{B}}{B} - \frac{\ddot{C}}{C} + \frac{\dot{A}}{A}\left(\frac{\dot{B}}{B} - \frac{\dot{C}}{C}\right) = 0 \quad (3.7)$$

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$$\frac{\ddot{A}}{A}-\frac{\ddot{B}}{B}+\frac{\dot{C}}{C}\left(\frac{\dot{A}}{A}-\frac{\dot{B}}{B}\right)=0 \qquad (3.8)$$

$$\frac{\ddot{A}}{A}-\frac{\ddot{C}}{C}+\frac{\dot{B}}{B}\left(\frac{\dot{A}}{A}-\frac{\dot{C}}{C}\right)=0 \qquad (3.9)$$

Equations (3.7) to (3.9) give us

$$\frac{B}{A}=n_1 \exp\left[m_1\int\frac{dt}{a^3}\right], \qquad (3.10)$$

$$\frac{C}{B}=n_2 \exp\left[m_2\int\frac{dt}{a^3}\right], \qquad (3.11)$$

$$\frac{A}{C}=n_3 \exp\left[m_3\int\frac{dt}{a^3}\right] \qquad (3.12)$$

Where m_1 m_2, m_3 and n_1, n_2, n_3 are constants of integration which satisfy the following relations

$$m_1+m_2+m_3=0, \qquad n_1n_2n_3=1 \qquad (3.13)$$

Using equations (3.10) to (3.12) we get

$$A=a\alpha_1 \exp\left[\beta_1\int\frac{dt}{a^3}\right] \qquad (3.14)$$

$$B=a\alpha_2 \exp\left[\beta_2\int\frac{dt}{a^3}\right] \qquad (3.15)$$

$$C=a\alpha_3 \exp\left[\beta_3\int\frac{dt}{a^3}\right] \qquad (3.16)$$

Where

$$\alpha_1=\left(\frac{1}{n_1^2n_2}\right)^{\frac{1}{3}}, \alpha_2=\left(\frac{n_1}{n_2}\right)^{\frac{1}{3}}, \quad \alpha_3=\left(n_1n_2^2\right)^{\frac{1}{3}} \qquad (3.17)$$

And

$$\beta_1=-\frac{2m_1+m_2}{3}, \quad \beta_2=\frac{m_1-m_2}{3},$$

$$\beta_3=\frac{m_1+2m_2}{3} \qquad (3.18)$$

Where

$$\alpha_1\alpha_2\alpha_3=1, \quad \beta_1+\beta_2+\beta_3=0 \quad (3.19)$$

4. IMPORTANT PHYSICAL QUANTITIES

The average scale factor a and volume scale factor V for the equation (3.1) are defined as

$$a=(ABC)^{\frac{1}{3}}, \qquad V=a^3=ABC \qquad (4.1)$$

The generalized mean Hubble parameter H is given by

$$H=\frac{1}{3}\left(H_x+H_y+H_z\right) \qquad (4.2)$$

Where $H_x=\frac{\dot{A}}{A}$, $H_y=\frac{\dot{B}}{B}$, $H_z=\frac{\dot{C}}{C}$ are the directional Hubble parameters in the directions of x, y and z axis respectively. The mean anisotropy parameter A is defined as

$$A=\frac{1}{3}\sum_{i=1}^{3}\left(\frac{H_i-H}{H}\right)^2 \qquad (4.3)$$

The expansion scalar θ and shear scalar σ^2 are defined as follows

$$\theta=v^i_{;i}=\frac{\dot{A}}{A}+\frac{\dot{B}}{B}+\frac{\dot{C}}{C} \qquad (4.4)$$

And

$$\sigma^2=\frac{1}{2}\sigma_{ij}\sigma^{ij}=\frac{1}{3}\left[\left(\frac{\dot{A}}{A}\right)^2+\left(\frac{\dot{B}}{B}\right)^2+\left(\frac{\dot{C}}{C}\right)^2-\frac{\dot{A}\dot{B}}{AB}-\frac{\dot{B}\dot{C}}{BC}-\frac{\dot{C}\dot{A}}{CA}\right]$$
(4.5)

Where

$$\sigma_{ij}=\frac{1}{2}\left(v_{i;\alpha}h^\alpha_j+v_{j;\alpha}h^\alpha_i\right)-\frac{1}{3}\theta h_{ij} \qquad (4.6)$$

$h_{ij}=g_{ij}-v_iv_j$ *is the proejction tensor.*

The deceleration parameter q is the measure of cosmic accelerated expansion of the universe. It is defined as

$$q=-\frac{\ddot{a}a}{\dot{a}^2} \qquad (4.7)$$

The nature of the universe models is determined with the help of the sign of q. The positive value of deceleration parameter gives a decelerating model while the negative value shows inflation. Since there are four field equations and five unknowns, so we use a well-known relation between the average scale factor a and average Hubble parameter H is as

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$$H = pa^{-s} \tag{4.8}$$

Where, p and s are positive constants

Using the equations (4.2) and (4.8), we obtain

$$\dot{a} = pa^{1-s} \tag{4.9}$$

and the deceleration parameter becomes

$$q = s - 1 \tag{4.10}$$

Integrating equation (4.9) yields

$$a = \left(spt + l_1\right)^{\frac{1}{s}}, \qquad s \neq 0 \tag{4.11}$$

and

$$a = l_2 \exp(pt), \qquad s = 0 \tag{4.12}$$

Where l_1 and l_2 are constants of integration. Thus we have two different models of universe corresponding to these values of the average scalar factor.

5. DIFFERENT TYPES OF UNIVERSES

(i) Singular model of the universe (when $s \neq 0$)

Here we discuss the model of the universe when $s \neq 0$ for the equation $a = \left(spt + l_1\right)^{\frac{1}{s}}$, $s \neq 0$ and we obtain the values of metric coefficients A, B and C are

$$A = \alpha_1 \left(spt + l_1\right)^{\frac{1}{s}} \exp\left[\frac{\beta_1 \left(spt + l_1\right)^{\frac{s-3}{s}}}{p(s-3)}\right], \quad s \neq 3 \tag{5.1}$$

$$B = \alpha_2 \left(spt + l_1\right)^{\frac{1}{s}} \exp\left[\frac{\beta_2 \left(spt + l_1\right)^{\frac{s-3}{s}}}{p(s-3)}\right], \quad s \neq 3 \tag{5.2}$$

$$C = \alpha_2 \left(spt + l_1\right)^{\frac{1}{s}} \exp\left[\frac{\beta_3 \left(spt + l_1\right)^{\frac{s-3}{s}}}{p(s-3)}\right] \quad s \neq 3 \tag{5.3}$$

The directional Hubble parameters H_i $\left(i = 1, 2, 3\right)$ becomes

$$H_i = \frac{p}{\left(spt + l_1\right)} + \frac{q_i}{\left(spt + l_1\right)^{3/s}} \tag{5.4}$$

The mean generalized Hubble parameter and volume scale factor are defined as

$$H = \frac{p}{spt + l_1}, \quad V = \left(spt + l_1\right)^{\frac{3}{s}} \tag{5.5}$$

And the mean anisotropy parameter becomes

$$A = \frac{\beta_1^2 + \beta_2^2 + \beta_3^2}{3p^2 \left(spt + l_1\right)^{\frac{(6-2s)}{s}}} \tag{5.6}$$

The expansion scalar θ and shear scalar σ for the model are expressed as

$$\theta = \frac{3p}{spt + l_1}, \qquad \sigma^2 = \frac{\beta_1^2 + \beta_2^2 + \beta_3^2}{2\left(spt + l_1\right)^{\frac{6}{s}}} \tag{5.7}$$

(ii) Non – Singular model of universe (when $s = 0$)

For this model, $s = 0$ and average scale factor $a = l_2 \exp(pt)$ gives the metric coefficients A, B, and C are defined as

$$A = \alpha_1 l_2 \exp\left(pt\right) \exp\left[-\frac{\beta_1 \exp(-3pt)}{3pl_2^3}\right], \tag{5.8}$$

$$B = \alpha_2 l_2 \exp\left(pt\right) \exp\left[-\frac{\beta_2 \exp(-3pt)}{3pl_2^3}\right], \tag{5.9}$$

$$C = \alpha_3 l_2 \exp\left(pt\right) \exp\left[-\frac{\beta_3 \exp(-3pt)}{3pl_2^3}\right] \tag{5.10}$$

The directional Hubble parameter H_i is given by

$$H_i = p + \frac{\beta_i}{l_2^3} \exp\left(-3pt\right) \tag{5.11}$$

The mean generalized Hubble parameter and volume scale factor are

$$H = p, \qquad V = l_2^3 \exp\left(3pt\right) \tag{5.12}$$

Also, The mean anisotropy parameter A, expansion scalar θ and shear scalar σ are defined as

$$A = \frac{\beta_1^2 + \beta_2^2 + \beta_3^2}{3p^2 l_2^6 \exp\left(6pt\right)}$$

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$\theta = 3p$

$$\sigma^2 = \frac{\beta_1^2 + \beta_2^2 + \beta_3^2}{2l_2^6 \exp(6pt)} \qquad (5.13)$$

6. CONCLUSION

In this paper we have discussed the current phenomenon of accelerated expansion of universe in the frame work of newly proposed f (R, T) theory of gravitation. Here we have taken $f(R,T) = R + 2\mu T$ and explore exact solution of Bianchi type I cosmological model and we have obtained two exact solutions with assumption of constant value of deceleration parameter and Hubble parameter further we have obtained the solution for two different models of universe namely

(i) The first solution gives a singular model with power law expansion and
(ii) Second solution gives a non singular exponential expansion of universe

And some physical parameters for the Bianchi type I cosmological model are also discussed.

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