IARJSET
ISSN (Online) 2393-8021
ISSN (Print) 2394-1588
International Advanced Research Journal in Science, Engineering and Technology
ISO 3297:2007 Certified
Vol. 3, Issue 12, December 2016

Finite Difference Modelling of Solar Thermal Powered Injera Baking Oven

Desta Goytom[1]

Faculty of Mechanical Engineering, Jimma Institute of Technology, Jimma, Ethiopia [1]

Abstract: This paper deals with simulation of transient heat transfer analysis of the solar powered injera baking system in which the heat transfer oil is heated using solar energy by parabolic trough and the oil circulates through the space below the baking -pan in the kitchen. This has been done by putting a glazed baking pan at the top of the storage tank which is in direct contact with the hot oil. The time history of heat up temperature of baking oven surface was simulated by Matlab Software for an algorithm has been developed using explicit finite difference approach. The simulation was done by varying the pan thickness for 10mm, 8mm and 5mm as well as the temperature of the hot oil in contact with the pan. The results had shown that for 10mm pan thickness, acceptable heats up times were recorded for the hot oil temperature values of 290 °C and 300 °C. Furthermore, at 8mm pan thickness the need for hot oil temperature was lowered to values ranging from 270°C to 300°C. But comparatively fast heats up times were recorded for a pan thickness of 5mm and hot oil temperature ranges from 255°C to the maximum value of 300°C.

Keywords: Baking pan, Storage tank, Simulation, Finite Difference method.

I. INTRODUCTION

Energy plays an important role on the development of a nation and development is possible through an increasing efficient use and extensive harnessing of various forms of energy. Despite rapid urbanization, the majority of Ethiopians still live in rural areas, and access to and utilization of energy resources varies considerably thorough the country. Even though Ethiopia has enormous potential for developing various energy resources, the per capita energy consumption remains to be among the lowest in the world [1].

Injera, a processed food which be made from cereals such as , teff, millet, sorghum, maize, wheat, rice etc., or combinations of those passed through fermentation and rigorous baking process, is the widely and cultural food of some east African countries particularly Ethiopia, Eritrea. Injera was baked most commonly on a clay plate called Mitad that is placed over a three stone stove or on specialized electric stove. When a fermented dough poured on a hot clay pan and stayed until the boiling temperature reached; bubbles from the boiling water escape forming thousands of tiny craters (eyes) that give the peculiar Injera texture. The traditional Mitad consists of a griddle plate of 'black' clay set on a base of stone and clay [2, 3].

Injera baking is the most energy intensive process because it requires a bulk of domestic energy demand. In most household's injera baking system is carried out using an open fire /three stone / baking systems as shown in figure 1 which is inefficient and energy wasteful technique. Injera baking requires temperatures ranging from 180 °C - 220 °C [4]. It is reported that cooking and baking account for over 50% of all primary energy consumption in the country. The major energy source for Injera baking is biomass. Hence intensive biomass utilization for Injera baking is accounted for deforestation, expensive fire wood price and poor kitchen environment [5]. This traditional biomass based cooking affected health, energy, school time, and hardship issues of women and children [6].

Introducing a new alternative energy source for baking injera is an important aspect from environmental and economic point of view. Solar thermal powered injera baking can benefit the environment by decreasing deforestation and the associated desertification. It can also decrease the health hazards associated with indoor fire cooking. Moreover, Women in villages and in some urban communities are relieved from economic burdens associated with firewood gathering or purchase. Thus to reduce and avoid the above mentioned problems, it is so significant to look for a new means of energy source to be utilized for backing injera. The use of solar energy for the purpose of cooking food presents a viable alternative to the use of fuel wood, kerosene, and other fuels traditionally used in developing countries.

Figure 1: open fire (three stone) injera baking system

DOI 10.17148/IARJSET.2016.31223

Converting the sun's radiant energy to heat is the most common and well developed solar energy conversion technology to alleviate the stated problems [7]. Among many application of solar energy, solar cooking is one which uses an arrangement of reflectors to concentrate solar energy on a cooking vessel so that solar energy will be changed in to heat energy for cooking. A number of innovative designs have been developed now a day and are being used in many parts of the world [8].

For a solar cooking system to be accepted and adopted in most of the households, the following objectives have to be satisfied [9].

- ✓ The cooking should be done without moving out of the kitchens.
- ✓ A reduction in the use of conventional energy.
- ✓ Cooking should be carried out at any time of day.
- ✓ Time taken for cooking must be comparable with conventional cooking.

To satisfy the above requirements related to solar cooking systems, a solar thermal powered indoor injera baking system is proposed where in the solar energy is transferred to the kitchen by means of a circulating heat transfer fluid and there is also heat storage.

Mondal and Datta (2010) developed a 2D CFD model for crust less bread baking to facilitate better understanding of the baking process. Simulation was done for heat and mass transfer from the bread during baking. They found that, the core temperature of the bread reached 95^{o}C at the end of baking, where moisture content of the bread complies with good quality bread. Purlis and Salvadori (2009b) predicted temperature and water content in the bread during baking. Finite element method was used based on a mathematical model considering moving evaporation front, evaporation condensation mechanism and crust development during baking. Another suggested hypothesis for porous-bodies is based on mathematical model proposed by Luikov (1975) to describe Simultaneous heat and mass transfer during drying and baking. This phenomenological approach applies the concept of irreversible thermodynamics and includes the effect of temperature on the water transport (i.e. thermo-diffusion). This model has been used as the basis of this study, but here based on thermo-physical properties of injera batter obtained from its composition (i.e., Carbohydrate, protein, fat, ash, fiber and water contents, temperature and density) [10].

The main objective of the study presented in this paper is to evaluate the performance the heat collecting elements (injera and the baking pan), using Matlab Software for an algorithm developed using explicit finite difference approach

II. DESCRIPTION OF SOLAR THERMAL POWERED INJERA BAKING SYSTEM

It uses parabolic trough solar collector to convert the solar radiation in to heat energy. The heat energy is conveyed from the collector to baking pan surface using heat transfer oil.

The solar powered injera baking oven has the following main components:-

- ✓ The parabolic through solar collector is used to collect and reflect the solar radiation and heat up the heat transfer oil in the receiver tube.
- ✓ Well insulated oil storage tank
- ✓ The baking pan assembly used for baking injera using the heat gained from solar system via heat transfer oil.
- ✓ The piping lines from the receiver tube to the oil gallery under the baking pan using oil pump to circulate the heat transfer oil through the system.

The operational principle solar thermal powered baking system as can be shown in the figure 2; the baking pan and the heat storage system are placed separately. Hence, Heat Is Transferred From The Heat Storage To The Baking Pan Assembly Indirectly Using Heat Transfer Fluid.

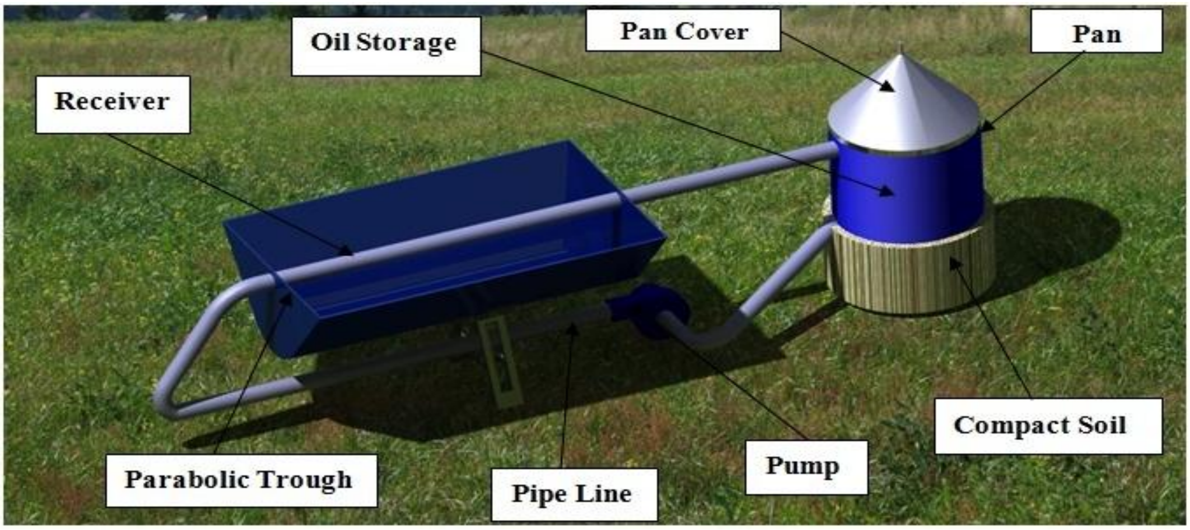

Figure 2: Schematic diagram of Solar powered Injera baking system.

DOI 10.17148/IARJSET.2016.31223

The Thermal oil (thermia oil B) was used as a working fluid medium since the injera baking process requires very high surface temperature usually 180°C to 220°C. The heat transfer fluid stored in the storage tank after which have been heating by solar thermal energy from parabolic receivers to oil, it will be pumped to the baking pan system. Accordingly heat would be transferred from the heated oil to the baking pan by two modes of heat transfer such as convection and conduction heat transfer. After delivering the thermal energy to the baking pan the oil from the well insulated oil storage tank to receiver through the return pipe line so as to get heated and this circulation will continue for the required period of time

UNSTEADY STATE HEAT BALANCE MODEL OF BAKING PAN

With the assumption of negligible heat loss due to the glazing material between the pan and the hot oil, neglecting losses through the left and right sides, and assume that the pan is left uncovered in its initial heating up condition. The transient heat transfer with in the pan can be expressed as:

$$\rho_p c_p\left(\frac{\partial T}{\partial t}\right) = h_{oc}(T_f - T_{bp}) - h_a\left(T_{sp} - T_a\right) + \sigma\varepsilon\left(T_{sp}^4 - T_a^4\right) \qquad (1)$$

Where: ρ_p density of the pan material (clay), (Kg/m^3)

c_p Specific heat capacity of clay, (KJ/Kg K)

k_p Conductivity of clay, (W/m^{2}K)

T_{sp} Upper Surface temperature of baking pan, (K)

T_{bp} Bottom surface temperature of baking pan, (K)

ε Emissivity of the pan

$\sigma = 5.67 \times 10^{-8}$ Stefan Boltzmann constant

The three right hand side terms represent convection between hot oil and the lower surface of the pan, convection between the plate upper surface and the surrounding air and radiation between the upper plate surface and the surrounding respectively.

Determination of the two convection heat transfer coefficients and the radiation heat transfer can be shown as follows;

Convection from the Heated Oil to the Baking Pan**:** To determine the convection heat transfer coefficient on the surface of the baking pan in contact with the hot oil. The relation for natural convection between a cold surfaces facing downward is given by [11]

$$N_{uL} = 0.27 \times R_{aL}^{1/4} \qquad 10^7 \leq R_{aL} \leq 10^{11} \qquad (2)$$

But the Rayleigh number of the flow is given by

$$R_{aL} = \frac{g\beta\left(T_{bp} - T_\infty\right)L^3}{\upsilon\alpha} \qquad (3)$$

Where: β volumetric expansion of the fluid.

g Gravitational acceleration

υ Kinematic viscosity

α Thermal diffusivity of the fluid

L is the characteristic length for cylindrical objects given by; $L = 0.9 \times D_p$ Where: D_p - the baking pan diameter

Finally the convection heat transfer coefficient from the relation:

$$h_{oc} = \frac{N_{uL}k}{L} \qquad (4)$$

Where: k is the conductivity of the fluid at the film temperature

$$T_{film} = \frac{T_f + T_{bp}}{2}$$

Convection from Baking Pan to the Atmosphere**:** For laminar flow, a flow less than the critical Reynolds number ($R_{eL} = 5 \times 10^5$), Air convection from a flat plate will be given as follows

$$N_{uL} = \frac{h_a L}{k} = 0.664 R_{eL}^{1/2} Pr^{1/3} \quad Pr \geq 0.6 \qquad (5)$$

$$R_{eL} = \frac{\rho_a v_\infty L}{\mu_a} \qquad (6)$$

Where: ρ_a The density of air, (kg/m^3)

v_∞ Air velocity at free stream, (m/s)

μ_a Dynamic viscosity of air, kgm^2/s^2

L The pan length, which is the pan diameter

All air properties are measured at film temperature, $T_{film} = \frac{T_a + T_{sp}}{2}$, where T_a is ambient air temperature and T_{sp} is the mean temperature of the plate surface. Finally the convection heat transfer coefficient is given by:

$$h_a = \frac{N_{uL}k_a}{L} \qquad (7)$$

Radiation from the Baking Pan

The radiation heat transfer from a flat top pan surface to ambient can be given by [11];

$$\dot{Q}_{net} = \varepsilon\sigma A_p\left(T_{sp}^4 - T_a^4\right) \qquad (8)$$

Where: A_p is upper surface area of the baking pan

The Heat up Period

The heat up temperature is the maximum temperature that is needed to start baking, about 220°C [12]. A simulation result done by using electric baking stove showed that during successive baking period the temperature will be lowered up to 75°C [13]. But during the gap between two Injera the surface temperature of baking pan was again raised up to a maximum of 185°C. This shows that once the maximum temperature (220°C) was achieved with a time range not far from the heat up period of conventional baking stoves (10 to 14 minutes), with constant heat supply a lot of Injera can be easily baked.

To achieve the required maximum temperature with some ranges of heat up time, the transient computational models developed in Eq. 1 to Eq. 8, were used to simulate for results.

For simulations a Matlab code is given below, as well as the input parameters used in Mathlab simulation are depicted in the following tables.

Table 1 Typical Design Data-Thermia B

Temperature °C	0	20	40	100	150	200	250	300	340
Density Kg/m^3	876	863	850	811	778	746	713	681	655
Specific Heat Capacity KJ/kg K	1.809	1.882	1.954	2.173	2.355	2.538	2.72	2.902	3.048
Thermal Conductivity W/m K	0.136	0.134	0.133	0.128	0.125	0.121	0.118	0.114	0.111
Prandtl No	3375	919	375	69	32	20	14	11	9

Table 2. The Standard Input Parameters used for the Simulation Runs

Parameters	**Values**
Baking Plate Material	Clay/Glue Mixture
Baking Plate Diameter	55cm
Pan Thickness	1cm,8mm and 5mm
Heat Capacity of Pan	880 J/K.kg
Conductivity of Pan	0.45W/m K
Convection Heat Transfer Coefficient Between the Pan and the Hot Oil	291.529W/m^{2}K
Air Convection Heat Transfer Coefficient at the Baking Surface	14.27 W/m^{2}K
Hot Oil Temperature in Contact with the Baking Pan	270, 280, 290, 300°C

Solar Contribution to the Baking Load

Solar thermal baking system requires high temperature of receiver out let fluid. The maximum solar contribution to the heating load is achieved during winter in the month of April and May and the minimum is at the month of July which is part of the summer season as shown in Fig 2. The system design was based on the average insolation value, November, so that the collector is expected to give the required amount of baking heat to the receiver fluid. Considering the months with comparatively lesser values from the average insolation the solar contribution to the baking load is reduced to 90% and the rest10% will be covered with the conventional source

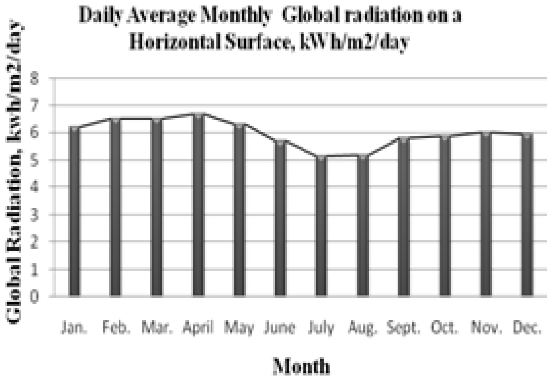

Figure 2. Daily Average Monthly Global Horizontal Radiation of the site

Development of Governing Equations and Boundary Conditions for Baking Pan

Discretization of the governing equations and the boundary conditions using one dimensional finite difference model as shown in Fig.3 can be expressed mathematically using explicitly finite difference.
Generally it will have three cases as internal node, first node and surface (nth) node

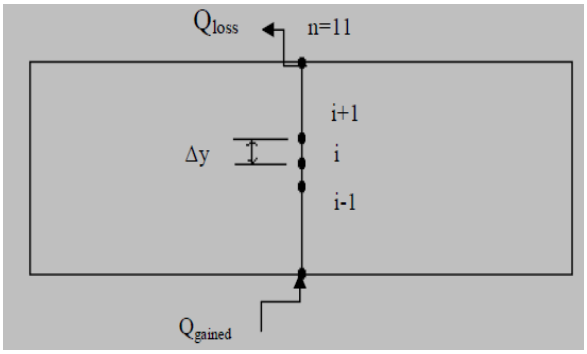

Figure 3. Discredited Pan Thickness Using Eleven Nodes

Internal Node: the internal node has direct interactions with both of its neighbouring nodes through conduction. The temperature of the node i at t+1 time step is explicitly given as;

$$T_i^{t+1} = \tau \times \left(T_{i-1}^{t} + T_{i+1}^{t}\right) + \left(1 - 2\tau\right) T_i^{t} \qquad (9)$$

Where $\tau = \frac{\alpha\Delta t}{\Delta y^2}$

$\alpha = \frac{k}{\rho c}$ Heat diffusivity

t : represents the time step

Δt Small time change of the total time for simulation

First Node**:** the first is in direct contact with the hot oil in the storage, neglecting the resistance of glazing, the heat balance for the first node is given by

$$h_{oc}\left(T_f - T_1^t\right) + k\frac{\left(T_2^t - T_1^t\right)}{\Delta y} = \rho\frac{\Delta y}{2}c_p\frac{\left(T_1^{t+1} - T_1^t\right)}{\Delta t} \qquad (10)$$

Where h_{oc} Hot oil to pan convection heat transfer coefficient

T_1 Temperature of the first node

T_f The hot oil temperature

Δy Differential thickness of pan

ρ Density of oil

Δt Differential time for computations

The temperature history of the first node can be expressed using explicit finite difference model is expressed by Eq. (11).

$$T_1^{t+1} = \left(1 - 2\tau - 2\tau\frac{h_{oc}\Delta y}{k}\right)T_1^t + 2\tau\tau_1^t + 2\tau\frac{h_{oc}\Delta y}{k}T_f \qquad (11)$$

The Top Surface (n^{th}) Node**:** assuming that during heating up period of the baking pan the upper surface is exposed to air convection, and the effect of radiation heat transfer is negligible.

The heat balance at the node can be written explicitly as

$$h_a\left(T_\infty - T_n^t\right) + k\frac{\left(T_n^t - T_{n-1}^t\right)}{\Delta y} = \rho\frac{\Delta y}{2}c_p\frac{\left(T_n^{t+1} - T_n^t\right)}{\Delta t} \qquad (12)$$

Where h_a Air convection heat transfer coefficient

T_∞ Ambient air temperature

And the explicit finite difference temperature notation for the surface node (T_n) can be expressed as

$$T_n^{t+1} = \left(1 - 2\tau - 2\tau\frac{h_a\Delta y}{k}\right)T_n^t + 2\tau\tau_n^t + 2\tau\frac{h_a\Delta y}{k}T_\infty \qquad (13)$$

The calculation of the convective heat transfer coefficients h_a and h_{oc} is based on the properties of air and hot oil at the corresponding film temperatures using Eq.7 and Eq. 4. The above three finite differences equations of nodes are used to simulate the temperature history of the baking surface for different changing parameters.

```
% USING EXPLICIT FINITE DIFFERENCE
APPROACH
%-----------------------------------------------------------------
------
%Imposing Material Properties and Initial Values
%-----------------------------------------------------------------
------
dx=0.001;        % Differential Length, in meters
nn=11;           % total Number of Nodes
k=0.45;              % Thermal Conductivity of the Baking
Pan, W/m K
cpc=880;         %Specific Heat Capacity of Pan, J/Kg K
rho=1460;        %Density of the Pan, Kg/m3
dt=1;            %Time Step
alpha=k/(rho*cpc);  %Thermal Diffusivity
tau=(alpha*dt)/(dx^2);
nts=3600;        %Number of Time Steps
Ta=25;           %Ambient Temperature
Ta1 = 300;       %Temperature of the Hot Oil
Ec=0.75;         %Emissivity of the Pan
sigma=5.67*(10^(-8));%Stefan Boltzmann Constant
hbot=291.529;         % Convection Coefficient Between
the Hot oil and the Pan
htop=14.27; ;       % Convection Coefficient Between the
air  and the Pan
z2=((2*htop*dt)/(rho*dx*cpc));
z=(2*htop*dt/(rho*dx*cpc));
z1=(2*sigma*Ec*dt/(rho*cpc*dx));
T=zeros(nn,nts+1);   %initializing the solution matrix.
time(1)=0;
time=zeros(1,nts+1);  %initializing the column vector
containing successive time

% -------------------------------------------------------------
%Imposition of Initial Condition
%-------------------------------------------------------------
fori=1:nn;
T(i,1)=Ta; %  The Initial Temperature of  Baking Pan
end

%-----------------------------------------------------------------
% Evaluation of Temperature at Each Nodes Explicitly
%-----------------------------------------------------------------
for j=2:nts+1;
time(j)=dt*(j-1)/3600;  %time given in hour
fori=2:nn-1;
  T(i,j)=tau*(T(i-1,j-1)+ T(i+1,j-1))+((1-2*tau)*T(i,j-1));
end

T(1,j)=(1-(2*tau)-((2*tau*hbot*dx)/k))*T(1,j-
1)+(2*tau*T(2,j-1))+((2*tau*hbot*dx)/k)*Ta1;
T(nn,j)=(1-(2*tau)-((2*tau*htop*dx)/k))*T(nn,j-
1)+(2*tau*T(nn-1,j-1))+((2*tau*htop*dx)/k)*Ta;
end
%---------------------------------------------------------------
%Ploting the Results for Surface and Center Nodes
%---------------------------------------------------------------
plot(time,T(1,:),'-r'); %plot for symmetric-center node of
the rod
```

```
hold on;
plot(time,T(11,:),'-black'); %plot for surface node
grid on;
xlabel('Time(hr)')
ylabel('Temprature( ^oC)')
title('Plot of Surface and Bottom Pan Temperatures vs Time step Using Explicit FDM ')
```

III.RESULTS AND DISCUSSION

The baking pan transient temperature profile for variable thickness of pan and temperature of hot oil will be discussed on the following sections. At a pan thickness of 1cm, the transient pan temperature plot is as shown in Fig.4.

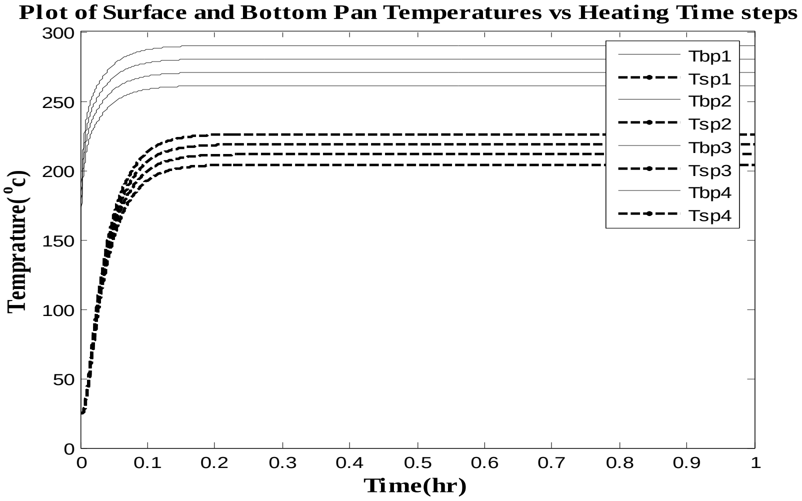

Figure 4. Temperature Profile of a 1cm thick Pan during Heating up period.

For 300oC hot oil temperature in contact with the baking pan, the time taken to raise the temperature of the pan surface to the required 220oC is about 9 minutes. Different heats up times obtained by varying the oil temperature as 290oC, 280oC and 270oC are listed in Table 3.

Table 3. Heating Time for Different Hot Oil Temperatures at 1cm Pan Thickness

Hot Oil Temperature (°C)	Pan Heating up Time (Hour)	T_{bp} (°C) (Bottom Pan Surface Temperature)	Tsp (°C) (Baking Pan Surface Temperature)
300	0.13	Tbp1=289	Tsp1=220
290	0.15	Tbp2=275	Tsp2=220
280	0.175	Tbp3=265	Tsp3=220
270	0.2	Tbp4=255	Tsp4=220

For 8mm pan thickness, the heats up periods were as shown in Fig. 5 and Table 4.

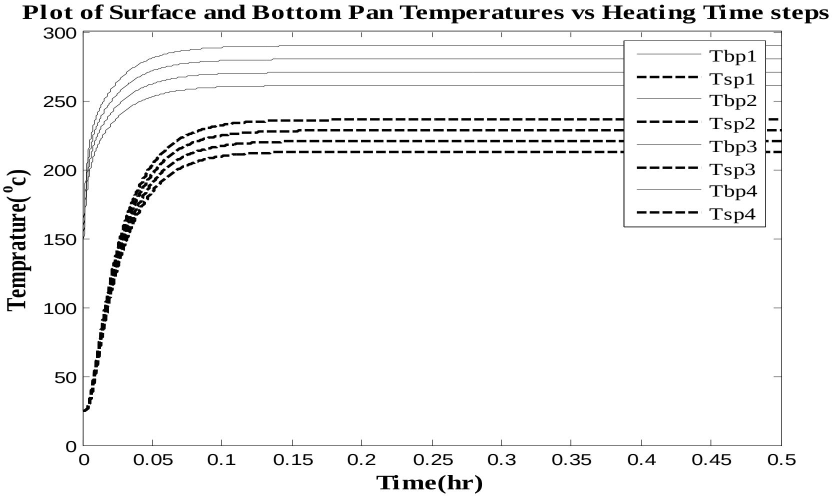

Figure 5. Temperature Profile of an 8mm thick Pan during Heating up Period

DOI 10.17148/IARJSET.2016.31223

At the pan thickness of 5mm, the time needed for the pan surface to reach the required maximum temperature is less as compared with that of the pan thickness of 8mm and 1cm. Fig. 6 describes the temperature profile of the baking pan at 5mm pan thickness. The results for each value of heated oil temperature are as listed in Table 5.

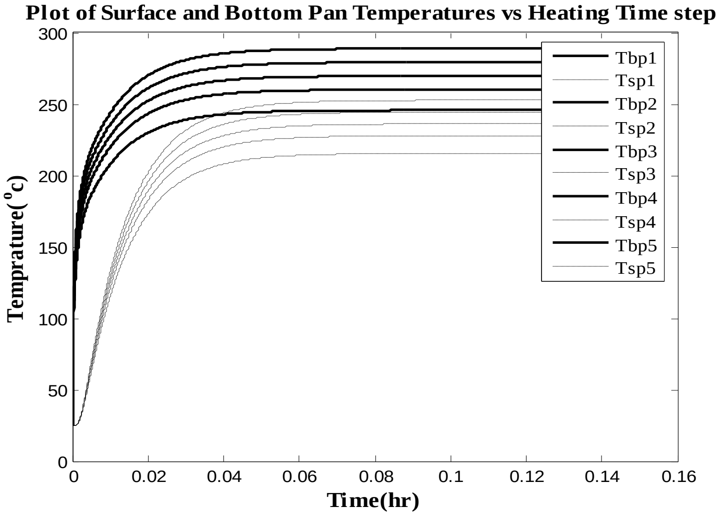

Figure 6. Temperature Profile of an 5mm thick Pan during Heating up Period

Table 4. Heating Time for Different Hot Oil Temperatures at 8mm Pan Thickness

Hot Oil Temperature (^{o}C)	Heating up Time (Hour)	Tbp (Bottom Pan Surface Temperature, ^{o}C)	Tsp (Baking Pan Surface Temperature, ^{o}C)
300	0.06	Tbp1=290	Tsp1=220
290	0.08	Tbp2=284	Tsp2=220
280	0.10	Tbp3=265	Tsp3=220
270	0.12	Tbp4=255	Tsp4=220

Table 5. Heating Time for Different Hot Oil Temperatures at 5mm Pan Thickness

Hot Oil Temperature (^{o}C)	Heating up Time (Hour)	Tbp (Bottom Pan surface Temperature,^{o}C)	Tsp(Baking Pan Surface Temperature, ^{o}C)
300	0.023	Tbp1=280	Tsp1=220
290	0.025	Tbp2=275	Tsp2=220
280	0.029	Tbp3=269	Tsp3=220
270	0.039	Tbp4=258	Tsp4=220
255	0.06	Tbp5=249.5	Tsp5=200

IV. CONCLUSION

The Finite Difference Method can possibly predict well the temperature distribution of the baking pan during initial heat up and cyclic baking of solar powered injera baking pan. From the simulation results of the baking pan, it can be concluded that the efficiency of the system can be increased with a decrease in pan thickness.
This effect can be seen clearly by comparing the results for cyclic baking (Fig. 4, 5 and 6). There is also significant reduction in unused period by reducing the thickness of the baking pan. Furthermore, increasing the number of injera baked per baking session is one way of improving the energy efficiency of the system. There is significant consumption of energy during heat up compared to the overall baking session. Generally, the proposed solar thermal powered baking pan gives reasonable heat up and baking time for 5 mm thick pan with heated oil temperature of 255^{0}C.

Finally, if it can increase the thermal conductivity of the pan and the thickness of the pan is reduced to a certain minimum value, the result of the current study has showed that there is a chance of replacing the receiver fluid from the expensive engine oil with water and steam.

ACKNOWLEDGMENT

The author acknowledges all the authors of references that have been given in this work.

REFERENCES

[1] Asres Welegiorgis, Solar Energy Assessment for an Application of Solar Pond at Lake Abijata: Ethiopia Rural Energy Development and Promotion Center, Addis Ababa, Ethiopia, 1987.
[2] http://www.ethiopianrestaurant.com/injera.html
[3] Hugh Burnham-Slipper BEng MPhil, Breeding a better stove, The university of Nottingham School of mechanical, materials and Manufacturing Engineering, December 2008.
[4] Hassen,A.A, Amibe, D.A., Nydal, O.J., Performance Investigation of Solar Powered Injera Baking Oven for Indoor Cooking: Proceedings of ISES Solar World Congress, Kassel, Germany, 2011
[5] World Bank report, Household Cook stoves, Environment, Health and Climate Change a new look at an old problem, 2011.
[6] J Parikh, Hardships and health impacts on women due to traditional cooking fuels: A case study of Himachal Pradesh-India, Elsevier International journal of Energy policy, 2011; 39: 7587–7594
[7] http://www.solarcooking, 2011. "Stichting Solar Cooking Nederland Baking Injera"
[8] PUR/PIR Manufactures, 2006. "Thermal insulation materials made of rigid polyurethane foam Properties" Report N°1; London
[9] Prasanna, U.R, Umanand, Optimization and Design of Energy Transport system for Solar Cooking Application: Applied Energy, vol. 88, 2011, pp.242-251
[10] Valentas,K.J., Rothstein,E., Singh, R.P., Hand book of Food Engineering Practice: CRC Press,LLC, Boca Raton, New York, 1997.
[11] Frank P.Incropera And David P.DeWitt, Fundamentals of Heat and Mass Transfer, Six Editions.
[12] Solar Cookers International (SCI), (www.solarcookers.org).
[13] Ezana Negusse and Robert Van Buskirk: Electric Enjera Cooker (Mogogo) Efficiency, Research Report: Energy Research and Training Division Department of Energy Ministry of Energy Mines and Water Resources Asmara Eritrea, October, 1996.
[14] Abdulkadir Aman Hassen and Demiss Alemu Amibe, finite element modeling of solar powered injera baking oven for indoor cooking