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# Centers in Subdivision and Inserted Graphs

**A. Anto Kinsley[1], J. Joan Princiya[2]**

Associate Professor, Department of Mathematics, St. Xavier's (Autonomous) College, Palayamkottai, India [1]

Research Scholar, Department of Mathematics, St. Xavier's (Autonomous) College, Palayamkottai, India [2]

**Abstract**: In this paper we study some concepts involving distance in subdivision graphs and inserted graphs and their centers. We prove some results on center, periphery and radius of the subdivision graph S (G) and inserted graph I(G)of a graph G for complete, cycle, complete bipartite, star and wheel graphs. An important theorem has been proved that in a connected graph the centers of G and S (G) have no common vertex. Graphs which are the periphery of some subdivision graph are characterized.

**Keywords**: Distance, Eccentricity, Radius, Diameter, Center, Subdivision graph and Inserted graph.

## I. INTRODUCTION

We consider ordinary graph G = (V, E), we mean a finite, undirected, connected graph without loop or multiple edges with vertex set $V_G$ and an edge set $E_G$ . A graph G with exactly one vertex is called a trivial graph, implying that the order of a nontrivial graph is at least 2. For a graph G and a pair u, v of vertices of G, the distance d (u ,v) between u and v is the length of a shortest u-v path in G. An u-v path of length d (u ,v) is an u-v geodesic in G. The degree of a vertex u, denoted by deg (u) is the number of vertices adjacent to u. A vertex of even degree is called an even vertex, while a vertex of odd degree is called an odd vertex. For a vertex v in a graph G, the eccentricity e(v) of v is the distance between v and a vertex farthest from v in G. Thus e (v) = max{d(u, v)| u$\epsilon$V}. The minimum eccentricity among the vertices of G is its radius and the maximum eccentricity is its diameter, which are denoted by rad(G) and diam(G) respectively. A vertex v in G is a central vertex if e(v)= rad(G) and a sub graph induced by the central vertices of G is the center and it is denoted by cen(G). If every vertex of G is a central vertex then cen(G)=G and G is called self- centered. A vertex v in G is a peripheral vertex if e (v)= diam (G) and a sub graph induced by the peripheral vertices of G is the periphery and it is denoted by per(G).

The subdivision graph of G is S (G)= (V $\cup$E,E$'$) where $E' = \{\{e, v\}: e \in E \text{ and } v \text{ is incident with } e\}$ . Each edge of G is replaced by a path of length 2. A vertex u of a graph G is called a universal vertex if u is adjacent to all other vertices of G.A graph can be constructed by inserting a new vertex on each edge of G and the resulting graph is called a box graph of G, denoted by B(G). For an edge e of G , $\bar{e}$ denotes the vertex of B(G) corresponding to the edge e. Let $I_G$ be the set of all inserted vertices in B(G). A graph I(G) with vertex set $I_G$ is called the inserted graph in which any two vertices are adjacent if they are joined by a path of length two in B(G).
Moreover, if

$$V_G=\{v_1, v_2,..,v_n\} \text{ and}$$

$$E_G=\{e_1, e_2,... e_m\}$$

then

$$V_{B(G)} = \{v_1, v_2, \dots, v_n, \overline{e_1}, \overline{e_2}, \dots, \overline{e_m}\}.$$

Example 1.1

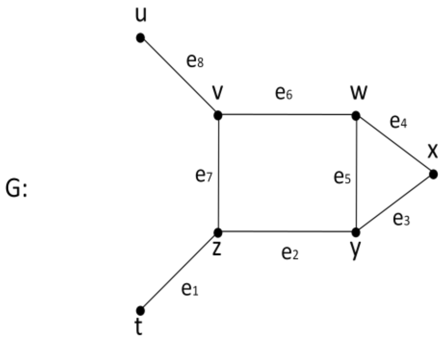


Figure 1 A graph G

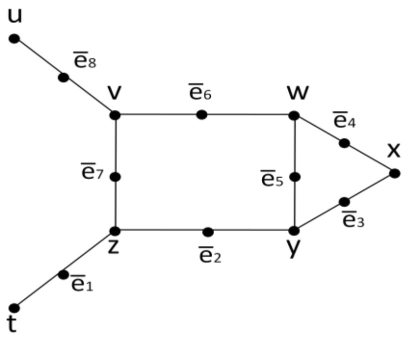


Figure 2 The box graph of a graph G

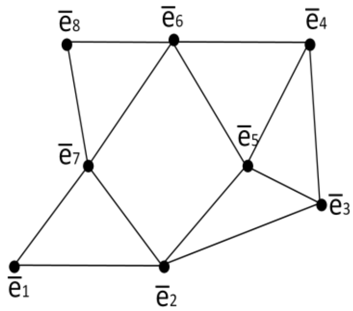


Figure 3 The inserted graph of a graph G

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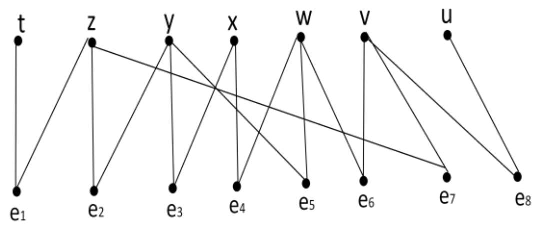


Figure 4 The subdivision graph of a graph G

## II. Main Results

Theorem 2.1
If G is a complete graph of order n >3, then C(S(G))=C(G) and per(S(G))=E(G).

Proof**:**
Let G be a complete graph $K_n$ and V(G)={$v_1$, $v_2$,..$v_n$}. If G is complete then it is clear that C(G)= V(G).Let S(G) be the subdivision graph of G. Then V(S(G))={$v_1$, $v_2$,...,$v_n$, $e_1$, $e_2$,... $e_m$} where m=n$C_2$.let $v_i$ , $v_j$ $\in$G $1\leq$ i, j $\leq$n be two adjacent vertices. Then there exist e such that e lies between $v_i$ and $v_j$ and so $d_G(v_i, v_j)$ = 1 . Now, by the definition of S(G), $d_{S(G)}(v_i, v_j)$ = 2. Hence $d_{S(G)}(v_i, v_j)$ = 2 $d_G(v_i, v_j)$. Also, there exist $e_i$ such that $e_i$ is adjacent to $v_j$ but not adjacent to a vertex $v_i$ in S(G).Therefore $d_{S(G)}(v_j, e_i)$ = 1. $d_{S(G)}(v_i, e_i)$= $d_{S(G)}(v_i, v_j)$ + $d_{S(G)}(v_j, e_j)$ =2+1=3 and so $e_{S(G)}(v_i)$=3. Let $e_j \in$S(G) be adjacent to the vertex $v_i$ in S(G). That is, $d_{S(G)}(v_i, e_j)$ = 1. $d_{S(G)}(e_i, e_j)$ = $d_{S(G)}(e_i, v_i)$ + $d_{S(G)}(v_i, e_j)$ = 4. Therefore, $e_{S(G)}(e_i)$ = $e_{S(G)}(e_j)$= 4.
Clearly, C(S(G))= {$v_1$, $v_2$,..$v_n$}. This implies, C(S(G))=V(G)=C(G) and Per(S(G)) = E(G) ={ $e_1$, $e_2$,... $e_m$}.

Note**:**
If G$\subseteq$ $K_n$ ,then C(G)$\subseteq$C(S(G)) and Per(G)$\subseteq$ per (S(G)).

Example 2.2

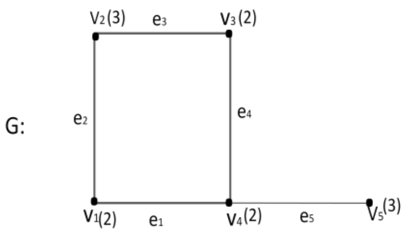


Figure 5 A graph G

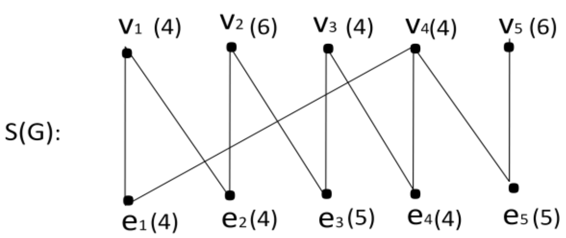


Figure 6 The subdivision graph of a graph G

Theorem 2.3
Let G be a cycle with at least three vertices. If G has a nontrivial center and a radius smaller than its subdivision graph, then S(C(G))is an induced sub graph of C(S(G)). Moreover, if G is self-centered then S(G) is also self-centered, and S(C(G))= C(S(G)) iff G is self-centered.

Proof**:**
Let r(S(G))= R+ n (n$\geq$2) where R is a radius of G. then for a vertex $\overline{uv}$ in S(C(G)) we have $e_G(u)$ = $e_G(v)$=R, which gives $e_G(uv)$ = $e_{S(G)}(\overline{uv})$ $\leq$ R +n= r(S(G)) and that is why $\overline{uv}$ is in C(S(G)). Moreover if G is self-centered then S(G) is an induced sub graph in C(S(G)), hence S(G) is self-centered. Further, if G is self-centered, then we have S(C(G))= S(G)=C(S(G)). Conversely suppose that, G is not self-centered. Then it contains an edge x y joining a central vertex x to a non-central vertex y, hence $e_G$ (x) = R and $e_G$ (y) = R +n. Then $e_G(xy)$ =$e_{S(G)}(\overline{xy})$$\leq$ R +n and $\overline{xy}$ is in S(C(G)).hence S(C(G))=C(S(G)) does not hold**.**

Theorem 2.4
If G is a cycle Cn , then C(S(G))= per(S(G)).
Proof**:**
Obviously the result is true. Since S(G) will form another cycle. ∴ C(S(G))= per(S(G)).

Theorem 2.5
For a complete bipartite graph G = $K_{n,n}$ , then r(S(G))=2r(G).

Proof:
Let G be a complete bipartite graph $K_{n,n}$ . Then r(G)=2=diam(G). Hence, C(G)= V(G). Now, V(G) $\cup$E(G) is the vertex set of S(G). let u and v be adjacent vertices of G, Then $d_G$(u, v)=1 and $d_{S(G)}$(u, v)=2 . let u and v be non-adjacent vertices of G, Then $d_G$(u, v)=2 and $d_{S(G)}$(u, v)=4 . Hence, $d_{S(G)}$(u, v)=2 $d_G$(u, v) . Let X and Y be the partition of V (G). Let v be the farthest vertex of u in G. Then u$\in$X implies v$\in$X and so $d_G$(u, v)=2 and $d_{S(G)}$(u, v)=4 . Hence, $e_G$ (u) =2and $e_{S(G)}$ (u)=4 . Therefore, r(G)=2 and r(S(G))=4.Hence, r(S(G))=2r(G).

Corollary 2.6
For a complete bipartite graph G= $K_{n,n}$ , then Per(S(G))=C(S(G)).

Proof:
Let G be a complete bipartite graph $K_{n,n}$ . Then r(G)=2=diam(G).Hence, C(G)= V(G)=Per(G). Using the above theorem, r(S(G))=4=diam (S(G)).Hence, C(S(G))= V(G)=Per(S(G)).

Theorem 2.7
Let G be a connected graph. Let e = u v be the only cut-edge and u, v be the only central vertices of G, then C(G) $\cap$C(S(G))=Φ

Proof**:**
Let G be a connected graph and e =u v is the only cut-edge of G. Then G-e contains two components $G_1$ and $G_2$ each of which contains vertices of C(G).let w be the vertex of G such that d(w, u)=e(u).and let $P_1$ be a w-u geodesic in


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G. at least one of contains $G_1$ and $G_2$ no vertices of $P_1$ ,say $G_2$ contains no vertices of $P_1$ . Let u be a central vertex of G, that belongs to $G_1$, then e(u)=r(G). This implies u$\epsilon$C (G). Let x be an eccentric vertex of v. d(x, v)=e(v).. .and $P_2$ be a v-x geodesic in G. let v be a central vertex of G, that belongs to $G_2$ , then e(v)=r(G). This implies v$\epsilon$C (G). Hence, u, v$\in$C(G) .we add an edge e (say $e_i$) to the central vertices u and v of G. Let V(G) $\cup$E (G) is the vertex set of S(G). Let $e_i$ be a central vertex of S(G). i.e., $e_i \in$ C(S(G)). Hence, C(G)$\cap$C(S(G))=$\Phi$

Theorem 2.8
Let $W_n$ , $n \geq 5$ be a wheel graph with the vertex set $\{v_1, v_2,..v_n\}$ and $v_n$ be the universal vertex. Then $C(S(W_n))=\{v_n\}=C(W_n)$ .

**Proof:**
Let G= $W_n$ , $n \geq 5$.let $V(G)=\{v_1, v_2,..v_n\}$. and $v_n$ is the universal vertex. Then e (v) =1= r(G) and so C(G)={ $v_n$ }. Let $\{v_1, v_2,...,v_n, e_1, e_2,... e_m\}$ be a vertex set of S (G). For every v$\epsilon$V(S (G)) e ($v_n$ )=3 and v$\epsilon$V(S (G)), e(v)=4 for $v \neq v_n$ and therefore C(S(G))={ $v_n$ }. Hence, C(G)=C(S(G))={ $v_n$ }.

Theorem 2.9
If G is a star graph $S_{1,n}$ , then I(G) is a complete graph $K_n$.

**Proof:**
Let G be a star graph and $\{v_1,v_2,..v_n\}$ be the vertex set of G. let $v_n$ be the universal vertex. Therefore, e($v_n$ )=1 and $e(v_1)=.....=e(v_{n-1})=2$. Let $\{v_1', v_2', \ldots v_m'\}$ be the vertex set of I(G). Then by the definition of I(G), we get every vertex of I(G) has eccentricity 1.

Hence distance between any two vertices is 1. That is, any pair of vertices are adjacent. Hence, I(G) is a complete graph with n vertices.

Theorem 2.10
Let G be a star graph and I(G) be a complete graph. then r(S(G))=2.r(I(G))

**Proof:**
Let G be a star graph. From theorem 2.9, I(G) is a complete graph. Let $\{v_1,v_2,..v_n\}$ be the vertex set of G. let $v_n$ be the universal vertex. Then r(I(G)) = r(G) =1. Also, $d_{S(G)}(v_n, v_i) = 2\, d_G(v_n, v_i)$ or $v_i \in V$.
Taking maximum on both sides, we have $e_{S(G)}(v_i)=2\, e_G(v_i)$. Hence, r(S(G)) =2.r(G) since r(I(G)) = r(G).

Theorem 2.11
A double star graph G has a cut-edge e = u v and u, v be the only central vertices of G, if each component of G-e is a star $G_1$ and $G_2$ , then $r(I(G_i))= r(G_i)$ ( i =1,2).

**Proof:**
Let G be a double star graph and e=u v be the only cut-edge of G. Then G-e contains two components $G_1$ and $G_2$. Let $G_1$ and $G_2$ be star graphs. Let u and v are universal vertices of $G_1$ and $G_2$ respectively. Then $r(G_1)= r(G_2)=1$. From theorem 2.9, $r(I(G_1))= r(I(G_2))$ = 1. Therefore, hence $r(I(G_i))= r(G_i)$ (i =1,2).

## III. CONCLUSION

Many researchers are concentrating the distance concept in graphs. In this paper we study the subdivision and inserted graphs for different types of graphs and investigate their properties. Many results have been found and compared for the above said graphs. In similar way the center concepts for other graphs can be studied.

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## BIOGRAPHY

**Dr. A. Anto Kinsley** has twenty nine years teaching experience in the department of Mathematics, St. Xavier’s College, Palayamkottai, Tamil Nadu and he has published twenty two research papers in both national and international journals. He has completed two UGC sponsored minor research projects. His area of specialisation is distance and algorithms in graphs. Ms. J. Joan Princiya is one of his research students. She is doing research in center concepts in graphs with respect to edges. She has published a research paper in an international journal.