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Literature Survey on High Resolution Direction of Arrival (DOA) Algorithms

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Abstract: This paper is an extensive study on realization of high resolution array based direction of arrival (DOA) estimation techniques which deals with beam forming. The techniques detailed include the conventional beam forming type of algorithms, maximum likelihood algorithms, the high resolution subspace based algorithms. The performance of MUSIC, ESPRIT, Minimum norm, and conventional beam forming techniques have been estimated using MATLAB. Based on the results realization of high resolution direction of arrival is achieved.

Keywords: direction of arrival; beam forming; high resolution; maximum likelihood; subspace.

I. INTRODUCTION

Direction of arrival (DOA) estimation or direction finding has been an active area of research for a long time. In radar application, they are useful for air traffic control and target acquisition [1]. Intelligence agencies use them for covert location of transmission and signal interception. Direction finding also finds application in position location and tracking systems. Most recently, direction of arrival estimation has become important in mobile radio communications. In a microwave receiver system, the angle of arrival (also referred as direction of arrival) information is extremely important. Since the DOA is obtained, from position of the emitter, this is the only parameter emitter cannot change easily. Thus the DOA becomes the most reliable sorting parameter in such receiving system.

There are seven ways to measure DOA, they are:

DOA is measured through narrow beam antenna and side lobe cancellation.
- DOA is measured through amplitude comparison.
- DOA is measured through phase comparison.
- DOA is measured through Doppler frequency shift.
- DOA is measured through TOA difference.
- DOA is measured through a microwave lens.
- DOA is measured through multiple beam arrays and beam forming networks.

In the most general sense, all non-rotating radio direction finding systems employ a DF antenna having an array of spatially-displaced aerials (also referred to as “elements”, three or more being required for non-ambiguous operation) that are illuminated by the received wavefront [2]. The resulting output voltages produced by these aerials exhibit characteristics (phase, amplitude, or both) that are then measured.

Since these characteristics are unique for every received azimuth in a properly designed DF antenna, the wavefront angle-of-arrival (bearing) can be ascertained by appropriately processing and analyzing the aerial output voltages.
To be somewhat more specific, modern non-rotating DF systems tend to fall into one of two broad categories. In phase-comparison DF systems [4][5], three or more aerials are configured in such a fashion that the relative phases of their output voltages are unique for every wavefront angle-of-arrival. Bearings can then be computed by appropriately analyzing the relative phases of these output voltages. Phase-comparison DF systems include pseudo-Doppler’s and interferometers.

In amplitude-comparison DF systems [3], [5], two or more directive antenna arrays are configured in such a fashion that the relative amplitudes of their outputs are unique for every wavefront angle-of-arrival. Bearings can then be computed appropriately analyzing the relative amplitudes of these output voltages. Amplitude-comparison DF systems include Watson- Watts and Wullenwebers.

Propagating fields are often measured by an array of sensors. A sensor array consists of a number of transducers or sensors arranged in a particular configuration. Each transducer converts a mechanical vibration or an electromagnetic wave into a voltage. Acoustic waves occur in microphone or sonar array applications. Mechanical waves are associated with seismic exploration and electromagnetic waves are used in wireless communications. Array signal processing applications include radar, sonar, seismic event prediction, microphone sensors, and wireless communication systems [6].

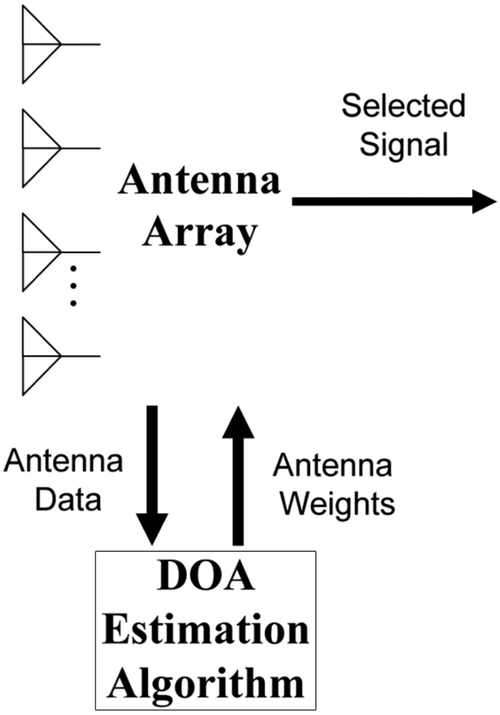

Fig.1. Antenna array and direction of arrival algorithms

In engineering applications, where an incoming wave is detected and/or measured by an array, the associated signals at different points in space can be processed to extract various types of information including their direction of arrival (DOA). Algorithms for estimating the DOA in antenna arrays are often used in wireless communications to increase the capacity and throughput of a network. In this paper, the focus will be on antenna arrays that receive or transmit electromagnetic waves in a digital communication network. Although most of the algorithms presented will focus on radio frequencies, we note that many of the discussed concepts can also be applied to mechanical and acoustic waves. We also note that the array processing algorithms presented can be used for real-time or offline applications.

DOA methods can be used to design and adapt the directivity of array antennas as shown in Figure 1. For example, an antenna array can be designed to detect the number of incoming signals and accept signals from certain directions only, while rejecting signals that are declared as interference.

DOA algorithms can be divided into three basic categories, namely, classical, subspace methods, and maximum likelihood (ML) techniques. In this book, the most important methods in each of these three categories will be discussed. The ML method offers high performance but is computationally expensive. The subspace methods also perform well and have several computationally efficient variants. The classical methods are conceptually simple but offer modest or poor performance while requiring a relatively large number of computations. Note that these algorithms are initially presented under the assumption that the signal sources are stationary in space and that the incoming signals are not correlated (no signals present due to multipath propagation).

This paper mainly deals with multiple beam arrays and beam forming network algorithms. Section II deals with conventional method of beam forming. Section III deals with maximum likelihood method. Section IV deals with high resolution direction finding algorithms which mainly deals with MUSIC, ESPRIT, ROOT MUSIC, Minimum norm methods. Section V gives the performance of the algorithms.

II. CONVENTIONAL METHOD

Conventional methods for direction-of-arrival estimation are based on the concepts of beamforming and null-steering, and do not exploit the nature of the model of the received signal vector x(k) or the statistical model of the signals and noise. Conventional techniques used for DOA estimation consists of electrically steering beams in all possible directions, and looking for peaks in the output power [8]. The conventional methods discussed here are the delay-and-sum method and the capon's minimum variance method. Figure below shows conventional beamforming system.

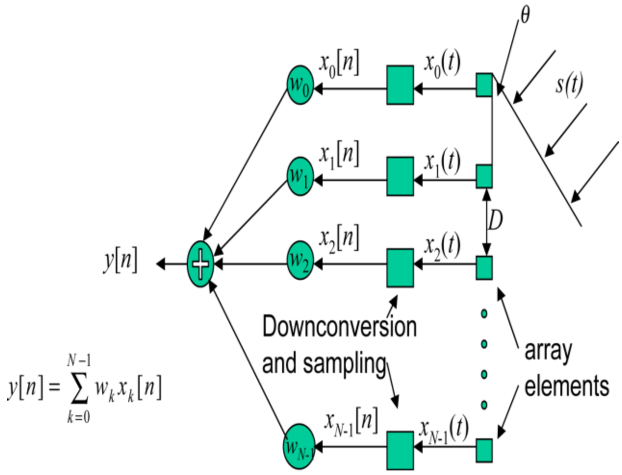

Fig.2.Narrowband beamforming system.

A. Delay and sum beamforming

The delay and sum method, also referred to as the classical beamformer method or Fourier method, is one of the simplest techniques for DOA estimation. Figure below shows the classical narrowband beamformer structure, where the output signal y(k) is given by a linearly weighted sum of the sensor element outputs. That is,

$$y(k) = w^H x(k) \qquad (1)$$

In the classical beamforming approach to DOA estimation, the beam is scanned over the angular region over the angular region of interest in discrete steps by forming weights w=a(θ) for different θ, and the output power at the classical beamformer as a function of the angle of arrival is given by

$$P_{cbf}(\theta) = a^H(\theta) R_{xx} a(\theta) \qquad (2)$$

Where R_{xx} is the autocorrelation matrix of the array input data and a(θ) is steering vector. The output power as a function of angles of arrival is often termed as the spatial spectrum. Clearly, the directions of arrival can be estimated by locating peaks in the spatial spectrum.

The delay and sum has many disadvantages. The width of the beam and the heights of the sidelobes limit the effectiveness when signals arriving from multiple directions and/or sources are present because the signals over a wide angular contribute to the measured average power at each look direction. Hence, this technique has poor resolution.

B. Capon's minimum variance method

Capon's minimum variance technique attempts to overcome the poor resolution problems associated with the delay-and-sum method. The idea is to use some of the degrees of freedom to form a beam in the desired direction while simultaneously using the remaining degrees of freedom to form nulls in the direction of interfering signals. The technique minimizes the contribution of the undesired interference by minimizing the output power while maintaining the gain along the look direction to be constant, usually unity.

$$min E[|y(k)|^2] = \min w^H R_{xx} w \qquad (3)$$

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$$subject\ to\ \ w^H a(\theta_0) = 1$$

Now the output power of the array as a function of the angle of arrival, using the capon's beamforming method, is given by the capon's spatial spectrum,

$$P_{capon}\,(\theta) = \frac{1}{a^H(\theta) R_{xx}^{-1} a(\theta)} \qquad (4)$$

By computing and plotting the capon's spectrum over the whole range of θ, the DOA's can be estimated by locating the peaks in the spectrum.

III.MAXIMUM LIKELIHOOD

Maximum likelihood (ML) techniques were one of the first techniques to be investigated for DOA estimation. Since ML techniques were computationally intensive, they were less popular than suboptimal subspace techniques. However, in terms of performance, the ML techniques are superior to the subspace methods, especially in low signal-to-noise ratio condition [9].

The maximum likelihood method estimates the DOA from a given set of array samples by maximizing the log-likelihood function. The likelihood function is the joint probability density function of the sampled data given the DOA's and viewed as a function of the desired variables which are the DOA's and viewed as a function of the desired variables which are the DOA's in this case. The operation principal of this method is to search for the directions that maximize the log of the likelihood function. The ML criterion signifies that plane wave from these directions are most likely to cause the given samples to occur [18] [19] [20].

The log likelihood function is given by

$$J = -NDlog\sigma^2 - \frac{1}{\sigma^2}\sum_{k=1}^{N} |x(k) - A(\theta)s(k)|^2 \qquad (5)$$

Where x=[x(1)…..x(N)] is the array data input vector matrix of dimension M x N, A(θ)=[a(θ_1),…..a(θ_D)] is the spatial signature matrix of the dimension M x D, S=[s(1),….,s(N)] is the signal waveform matrix of dimension D X N, and N=[n(1),…..,n(N)] is the noise matrix of the dimension M x N. σ^2 is an unknown scalar .

To compute the maximum likelihood estimator, the log likelihood function of has to be maximized with respect to the unknown parameters. This yields the following maximization problem:

$$(\theta, S)^{\max} \{-NDlog\,[\tfrac{1}{ND}\sum_{k=1}^{N} |x(k) - A(\theta)s(k)|^2]\} \qquad (6)$$

The logarithm being a monotonic function, maximum is equivalent to the following minimization problem:

$$(\theta, S)^{\min} \{\sum_{k=1}^{N} |x(k) - A(\theta)s(k)|^2\} \qquad (7)$$

IV.HIGH RESOLUTION SUBSPACE METHODS

There are some fundamental limitations in resolution in classical beamforming methods even though they are often successful and widely used. Most of these limitations arise due to the fact they do not exploit the structure of the narrowband input data model of the measurements. Schmidt [10] and Bienvenu and Kopp [11] were the first to exploit the structure of a more accurate data model for the case of sensor arrays of arbitrary form.

The technique proposed by Schmidt is called the multiple signal classification (MUSIC) algorithm, and has been thoroughly investigated since its inception [12] [13] [14]. Apart from MUSIC, the primary contributions to the subspace-based algorithms include the estimation of signal parameters via rotational invariance technique (ESPRIT) proposed by ROY et. Al.,[15][16][17],

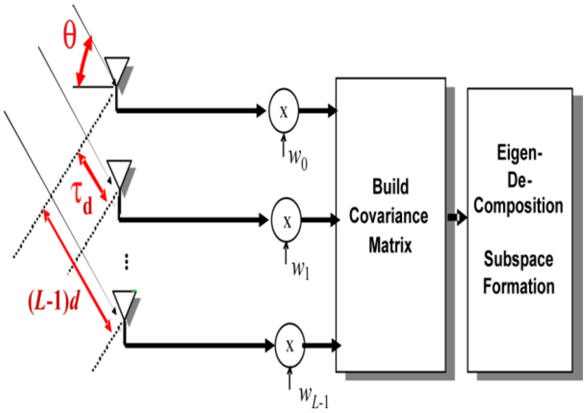

Fig.3. Eigendecomposition of antenna array signals. θ is the angle of arrival; *D* is the distance between two adjacent elements in meters; τ_d is the time delay of arrival between two successive elements in seconds; and there are *L* elements in the array.

A. MUSIC

Consider an array of M antenna elements receiving a set of plane waves emitted by P (P < M) sources in the far field of the array. We assume a narrow-band propagation model, i.e., the signal envelopes do not change during the time they take for their wave fronts to travel from one sensor to another. Suppose that the signals have a common frequency of f_o; then, the wavelength is given by $\lambda = c/f_o$ where c is the speed of propagation. The received M-vector **r**(t) at time t is.

$$r(t)= A*s(t) + n(t) \qquad (8)$$

$s(t)=[s_1(t)\ s_2(t).\ .\ .\ .\ .\ .\ s_P(t)]^T$ is a P vector source $A=[a(\theta_1)\ \ a(\theta_2)\ .\ .\ .\ .\ a(\theta_P)]$ is a steering matrix of size M X P. In which $a(\theta_i)$ is the i^{th} steering vector response of the array to the i^{th} source arriving from θ_i direction. $n(t)=[n_1(t)\ \ n_2(t)\ .\ .\ .\ .\ .\ n_m(t)]^T$ is the additive noise process.

Source signals are statistically independent & partially correlated or completely correlated. The array may have the arbitrary shape and response. The noise process is independent of the source, having zero mean and it may be either white noise or colored noise with distribution unknown. Covariance matrix R is given by

$$R = \in \{r(t) * r(t)^H\} = AR_sA^H + R_n \qquad (9)$$

Where $R_s = \in \{s(t) * s(t)^H\}$ is pxp signal covariance matrix. $R_n = \in \{n(t) * n(t)^H\}$ is mxm noise covariance matrix. If the noise is white then $R_n = \sigma^2 I$. But, in reality the correlation matrix is approximated by uniform averaging by some number of snapshots as

$$R = \frac{1}{snaps\ hots} \sum_1^{snaps\ hots} r(t) * r(t)^H \qquad (10)$$

Music (MUltiple SIgnal Classification) is a search base method of noise subspace in which noise information is retained based on property that steering vectors are orthogonal to any linear combination of noise subspace eigenvectors.

Principle of search based method, the array manifold is assumed to be known, and the arrival angles are estimated by locating the peaks of the function $S(\theta)=1/a(\theta)^H N\ a(\theta)$ where **N** is a matrix formed using the noise space eigenvectors. The steering vectors corresponding to incoming signal lies in the signal subspace are therefore orthogonal to noise subspace. In this method the estimates of DOA's is to search through a set of all possible steering vectors and find those that are orthogonal to noise subspace. If $a(\theta)$ is steering vector corresponding to one of the incoming signals then it should satisfy $a(\theta)^H * N=0$. Where N is the matrix formed using noise subspace eigen vector. In practice $a(\theta)$ will not be orthogonal to noise subspace due to errors in estimation of N. However, the power spectrum is given as

$$S_{music}(\theta) = \frac{1}{a(\theta)^H * N * a(\theta)} \qquad (11)$$

$$N = \sum_{i=P+1}^{M} e_i\, e_i^H \qquad (12)$$

e_i = eigen vector corresponding minimum value of R.

B. ESPRIT

ESPRIT is an algebraic subspace DOA estimation method. Algebraic methods do not require search procedure. ESPRIT achieves a reduction in computational complexity by imposing a constraint on the structure of an array. The ESPRIT algorithm assumes that an antenna array is composed of two identical sub arrays. The sub arrays may overlap, that is, an array element may be a member of both sub arrays.

If there are a total of M elements in an array and m elements in each subarray, the overlap implies that $M \leq 2m$. For subarrays that do not overlap, $M = 2m$. The individual elements of each subarray can have arbitrary polarization, directional gain, and phase response, provided that each has an identical twin in its companion subarray[23][24]. Elements of each pair of identical sensors, or doublet, are assumed to be separated physically by a fixed displacement (translational) vector. The array thus possesses a displacement (translational) invariance (i.e., array elements occur in matched pairs with identical displacement vectors). This property leads to the rotational invariance of signal subspaces spanned by the data vectors associated with the spatially displaced subarrays; the invariance is then utilized by ESPRIT to find DOAs.

The two subarrays , array-1 and array-2 are displaced by distance 'd'. The signals induced on each of the arrays are given by

$$r^1(t)=A*s(t)+n_1(t) \qquad (13)$$

$$r^2(t)=A*\Phi*s(t)+n_2(t) \qquad (14)$$

where $r^1(t)$ and $r^2(t)$ are measurements due to displacement where

$$\Phi=\text{diag}[e^{-j2\pi\frac{d}{\lambda}\sin\theta_1}, \dots\dots, e^{-j2\pi\frac{d}{\lambda}\sin\theta_p}]. \qquad (15)$$

Let auto covariance matrix of $r^1(t)$ be $R^{11}=AR_sA^H+R_{n11}$ cross covariance matrix of $r^2(t)$ be $R^{21}=A\Phi R_sA^H+R_{n21}$ Where R_s covariance matrix compounds to source.

$$\widehat{R^{11}} = R^{11} - R_{n11} \qquad (16)$$

$$\widehat{R^{21}} = R^{21} - R_{n21} \qquad (17)$$

Find singular values of α_i from the equation

$$\widehat{R^{11}} - \alpha_i\widehat{R^{21}}, \text{ i=1,2,....,p} \qquad (18)$$

From the singular values one can find $\alpha_i = e^{j2\pi\frac{d}{\lambda}\theta_i}$

The assumptions made in the ESPRIT are noise is spatially and temporally white in nature and covariance matrices R_{n11} & R_{n21} are known in advance.

C. ROOT MUSIC

In the case of Uniform Linear Array (ULA), the scanning for DOA can be transformed into solving the roots of a corresponding polynomial. Root-MUSIC solves the rooting problem of a polynomial rather than finding the spectral peaks in the MUSIC algorithm. After lots of research and simulation, it is proved that Root-MUSIC has a better property than spectral MUSIC in some cases [22], such as resolution ability. The pre-process of Root-MUSIC is the same with MUSIC and the only difference between Root-MUSIC and MUSIC is the Direction Finding method.
From MUSIC algorithm, we can get

$$P_{MU}(\theta) = \frac{1}{a(\theta)^T E_N E_N^T a(\theta)} \qquad (19)$$

Which is used to scan by degree. However, for the moment, if we restrict our attention to uniform linear arrays with inter element spacing d, so that the i[th] element of a(θ) may be written as:

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$$a_i(\theta) = e^{j2\pi\frac{d}{\lambda}sin\theta} \quad \text{i=1, 2, …., M} \qquad (20)$$

Let us restrict our attention to the denominator $p_{MU}^{-1}(\theta)$, it may be written as:

$$p_{MU}^{-1}(\theta) = a(\theta)^T E_N E_N^T a(\theta)$$

$$\sum_{i=1}^{M}\sum_{k=1}^{M} e^{-j2\pi(d/\lambda)sin\theta} E_N E_N^T e^{-j2\pi k(d/\lambda)sin\theta} =$$

$$\sum_{l=-M+1}^{M-1} E_1 e^{-j2\pi l(d/\lambda)sin\theta} \qquad (21)$$

Where E_1 is the sum of entries of $E_N E_N^T$ along the i^{th} diagonal

$$E_1 = \sum_{i-k=l} E_N E_N^T \qquad (22)$$

If we define the polynomial P(z) as:

$$P(z) = \sum_{l=-M+1}^{M+1} E_l z^{-l} \qquad (23)$$

On the unit circle, evaluating the spectrum $P_{MU}(\theta)$ is equivalent to evaluating the polynomial P(z). we can use the roots of P(z) for direction of arrival estimation rather than scanning for peaks in $P_{MU}(\theta)$[21]. Definitely, peaks in $P_{MU}(\theta)$ are due to roots of P(z) lying close to the unit circle. Take the pole of P(z) at z_1 for example

$$z_1 = |z_1| e^{jarg\,(z_1)} \qquad (24)$$

It will result in a peak in $P_{MU}(\theta)$ at:

$$\sin(\theta) = \frac{\lambda}{2\pi d} \arg(z_1) \qquad (25)$$

Therefore, after solving the polynomial P(z), we can get D roots which locate near the unit circle mostly. Then, based on the relationship between z and θ, direction of arrival can be found.

D. MINIMUM NORM

The minimum norm method was proposed by Kumaresan and Tufts [26]. This method is applied to the DOA estimation problem in a manner similar to the MUSIC algorithm. The minimum norm vector is defined as the vector lying in the noise subspace whose first element is one having minimum norm [25]. This vector is given by:

$$g = \begin{bmatrix} 1 \\ \hat{g} \end{bmatrix} \qquad (26)$$

Once the minimum norm vector has been identified, the DOAs are given by the largest peaks of the following function [25]:

$$P_{MN}(\theta) = \frac{1}{\left|a^H(\theta)\begin{bmatrix} 1 \\ \hat{g} \end{bmatrix}\right|} \qquad (27)$$

The objective now is to determine the minimum norm vector g. Let **Q***s* be the matrix whose columns form a basis for the signal subspace. **Q***s* can be partitioned as [25]:

$$Q_s = \begin{bmatrix} \propto^* \\ \overline{Q_s} \end{bmatrix} \qquad (28)$$

Since the vector **g** lies in the noise subspace, it will be orthogonal to the signal subspace, **Q***s*, so we have the following equation [25]:

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$$Q_s^H \begin{bmatrix} 1 \\ \hat{g} \end{bmatrix} = 0 \quad (29)$$

The above system of equations will be underdetermined; therefore we will use the minimum Frobenius norm [25] solution given by:

$$\hat{g} = -\overline{Q_s}(\overline{Q_s^H Q_s})^{-1} \propto \quad (30)$$

Therefore

$$\hat{g} = -\overline{Q_s} \propto \Big/ (1 - \|\propto\|^2) \quad (31)$$

Once **g** has been computed, the Min-Norm function given above is evaluated and the angles of arrival are given by the *r* peaks.

E. COMPARISION

Algorithm	Consistency	Coherent Signal	Accuracy	resolution	Computation efficiency
music	Yes	No	Exact	Good	Good
Root-music	yes	Yes	Good	Exact	low
ESPRIT	yes	yes	Good	Exact	Efficient
Min-norm	yes	yes	low	good	low

All the methods discussed under subspace technique are consistent. Definitely, the MUSIC algorithm estimate DOA by scan incident angle one by one, and the step of scanning decide the estimation accuracy. In Min-Norm by using the minimum noise vector which lie in noise subspace we estimate the direction of arrival. However, ESPRIT and Root-music algorithm can compute the incident angle directly by subspaces and polynomial respectively for all consistent DOA.

Coherent signal are referred to signals which are greatly correlated with each other. When the coefficient of correlation is equal to 1, we define signals as coherent signals. It is well accepted that MUSIC algorithm is the first Significant and classical DOA estimation algorithm. But one of its most serious problems is solving coherent signals.

The accuracy is the most effective evidence to judge an algorithm. It is fair to say these four methods have good accuracy. In detail, some past simulation results of music, Min-Norm, root-music and ESPRIT algorithms show that their performance improves with more elements number of the array elements, with larger signal noise ratio(SNR), with larger snapshots of signals and greater angular separation between the signals. These improvements can be seen in form of sharper peaks in the MUSIC simulation and smaller errors in angle detection in the ESPRIT and ROOT-MUSIC simulation. However, it is said that there are more errors in DOA estimation by using ESPRIT algorithm compared to the MUSIC algorithm. Which means music is more accurate than ESPRIT. And min norm is inferior to ESPRIT.

The resolution is defined as the ability to distinguish two or more sources with the same of similar incident angle. As for MUSIC algorithm, the resolution ability is another weakness. It is not hard to understand that we cannot decide the exact number of signal from one peak in the graph of MUSIC algorithm. A small step of scanning can improve the resolution ability but cannot solve this problem totally. However, ESPRIT and Root-MUSIC have excellent resolution

The computation efficiency is defined as the amount of calculation in a particular DOA estimation work. Definitely, the greatest improvement of ESPRIT is in the area of computation efficiency. A large number of researches prove that ESPRIT has better computation efficiency than MUSIC. The computation efficiency of Root-music and min norm are worst in these four algorithms.

V. PERFORMANCE

The Performance of the some of the high resolution algorithm and beamforming algorithm is illustrated in the below table. Where three signals impinging the array of length 5 at an angles -30^0, -15^0, 40^0.

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Input direction	MUSIC	ESPRIT	MIN norm	MVDR	DAS
-30^{o}	-30^{0}	-30.72^{0}	-31^{0}	-22^{0}	-22.3^{0}
-15^{0}	-15^{0}	-14.8^{0}	-13.5^{0}	-22^{0}	-22.3^{0}
40^{0}	40^{0}	40.10^{0}	39^{0}	40.1^{0}	39^{0}

Form the above table we can conclude that when two sources are near to each other conventional beamforming techniques cannot identity these two sources. If two sources have minimum separation in their direction they can be identified.

VI. CONCLUSION

High resolution subspace DOA algorithms have high resolution and accuracy when compared with the conventional type of beamforming techniques like delay and sum and MVDR. Maximum likelihood estimator performs better than the subspace methods, but at the cost of increasing the computational complexity. Even though Min-Norm technique is generalized as high resolution technique it is inferior to MUSIC and ESPRIT.

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