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Unsteady MHD Free Convection Flow past a Vertical Porous Plate Considering Radiation and Volume Fraction Effects in a Nanofluid

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Abstract: This paper investigates the unsteady magneto hydrodynamic free convection flow past a vertical porous plate considering radiation and volume fraction effects in a nanofluid. It is assumed that the plate oscillate with constant frequency in time. The transformed coupled, dimensionless partial differential equations are solved numerically by using finite element method. The influence of physical parameters on velocity, temperature profiles are discussed and depicted with the aid of graphs. Finally the numerical values of skin friction and Nusselt number with in the boundary layer are compared to ensure the correctness of this numerical scheme. This investigation has great practical importance in many branches of Science, Engineering and Technology viz. cooling applications, material processing, biomedical application s and other possible areas.

Keywords: Nanofluid, Volume fraction, Radiation, MHD, FEM.

I. INTRODUCTION

A Nanofluid is the term coined by Choi [1] to describe this new class of nanotechnology-based heat transfer fluids that exhibit thermal properties superior to those of their host fluids or conventional suspension of particles in fluids.Nanofluid is a sold-liquid mixture which consists of nano particles and a base liquid. Nanoparticles are basically metals (Cu, Al, etc..),Oxides(Al_2O_3, T_iO_2), Carbides(SiC), Nitrites (SiN, AlN)or alternatively non-metals (graphite, carbon nano tubes-CNTs) and base fluid is usually a conductive fluid such as water or an ethylene glycol mixture and also other base fluids like mineral oil,engine oil, lubricants, bio-fluids, poor heat transfer fluids. The study of thermal properties of nanofluid has gained a lot of importance due to its potential applications in electronics, communication, defense, material synthesis and biomedical science. Several investigators used the nanofluid model proposed by Buongiorno[2] in which the author reported seven slip mechanisms take place in convective transport in nanofluids. However in this article the nanofluid model proposed by Tiwari and Das [3] is considered.

In many areas of engineering such as Nuclear reactor cooling system, Space technology, etc…the effects of thermal radiation in porous medium has major importance. Several authors investigated the effects of thermal radiation. Chamkha[4] studied the radiation effects on mixed convection over a wedge with a nanofluid. The effects of thermal radiation on boundary layer flow have also been considerably investigated by Ellahi[5] and Makinde et al.[6]. Finally, Sheikholeslami et al. [7] investigated magnetic field effect on nanofluid flow and heat transfer in a semi-annulus enclosure by considering the effects of thermophoresis and Brownian motion to get the gradient of nanoparticles volume fraction.

Novelty of this paper is the unsteady magnetohydrodynamic free convection flow past a vertical porous plate considering radiation and volume fraction effects in a nanofluidis studied by extendingthe problem investigatedin Hamad and pop [8]with consideration ofradiation and permeability terms. In this paper formation of the problem and numerical procedure are presentedin sections 2and3. Section 4 and 5 contains Grid independence study and validation of the code respectively, section 6 contains results and discussion. Finally section7 highlights the important conclusions from the present study.

II. MATHEMATICAL FORMATION OF THE PROBLEM

The Cartesian coordinate system $(\bar{x}, \bar{y}, \bar{z})$ and mathematical modeling of the problem under consideration is shown in Fig. 1. Flow direction is along the vertical porous plate which is the direction of $\bar{x}-axis$ and is perpendicular to $\bar{z}$ - axis . The entire system is rotating about $\bar{z}$ - axis with constant velocity Ω . Choose an unsteady free convective

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flow along the semi-infinite oscillatory (with constant frequency on $\bar{t}$) plate embedded in a nanofluid. Along the $\bar{z}$ - axis an external magnetic field B_0 is acting which is greater than the induced magnetic field and no electric field is applied so that oscillating plate can have small Reynolds number. The electrically non conducting plate produces J_z to be constant due to current density conservation equation. $\nabla . J = 0$ It is further assumed that surface temperature $\bar{T}_w$ relatively more than the ambient temperature $\bar{T}_\infty$ and all physical quantities depends on variables $\bar{z}$ and $\bar{t}$ .With the consideration of Tiwari and Das model, under the usual boundary layer and Boussinesq approximations are:

$$\frac{\partial \bar{w}}{\partial \bar{z}} = 0 \qquad (1)$$

$$\frac{\partial \bar{u}}{\partial \bar{t}} + \bar{w}\frac{\partial \bar{u}}{\partial \bar{z}} - 2\Omega \bar{v} = \frac{1}{\rho_{nf}}\left\{ \mu_{nf}\frac{\partial^2 \bar{u}}{\partial \bar{z}^2} + (\rho\beta)_{nf}\, g(\bar{T} - \bar{T}_\infty) - \sigma B_0^2 \bar{u} - \frac{\nu_f}{k}\bar{u} \right\} \qquad (2)$$

$$\frac{\partial \bar{v}}{\partial \bar{t}} + \bar{w}\frac{\partial \bar{v}}{\partial \bar{z}} + 2\Omega \bar{v} = \frac{1}{\rho_{nf}}\left\{ \mu_{nf}\frac{\partial^2 \bar{v}}{\partial \bar{z}^2} - \sigma B_0^2 \bar{v} - \frac{\nu_f}{k}\bar{v} \right\} \qquad (3)$$

$$\frac{\partial \bar{T}}{\partial \bar{t}} + \bar{w}\frac{\partial \bar{T}}{\partial \bar{z}} = \alpha_{nf}\frac{\partial^2 \bar{T}}{\partial \bar{z}^2} - \frac{1}{(\rho C_p)_{nf}}\frac{\partial q_r}{\partial \bar{z}} - \frac{Q_H}{(\rho C_p)_{nf}}(\bar{T} - \bar{T}_\infty). \qquad (4)$$

And the associated initial and boundary conditions on the vertical surface and in free stream can defined as.

$$\left.\begin{array}{ll} \text{for } \bar{t} < 0 & \{\forall\ \bar{z} \quad \bar{u}(\bar{z},\bar{t}) = 0, \quad \bar{v}(\bar{z},\bar{t}) = 0,\ \bar{T} = T_\infty \\ \text{for } \bar{t} \ge 0 & \begin{cases} \bar{z} = 0 & \bar{u}(0,\bar{t}) = U_0\left[1 + \frac{\varepsilon}{2}(e^{\text{int}} + e^{-\text{int}})\right], \\ & \bar{v}(0,\bar{t}) = 0,\ \bar{T}(0,\bar{t}) = \bar{T}_\infty \\ \bar{z} \to \infty & \bar{u}(\infty,\bar{t}) \to 0, \bar{v}(\infty,\bar{t}) \to 0,\ \bar{T}(\infty,\bar{t}) \to \bar{T}_\infty \end{cases} \end{array}\right\} \qquad (5)$$

$$\left.\begin{array}{l} \mu_{nf} = \frac{\mu_f}{(1-\phi)^{2.5}},\ \alpha_{nf} = \frac{K_{nf}}{(\rho C_p)_{nf}}, \rho_{nf} = (1-\phi)\rho_f + \phi\rho_s, \\ (\rho C_p)_{nf} = (1-\phi)(\rho C_p)_f + \phi(\rho C_p)_s \\ (\rho\beta)_{nf} = (1-\phi)(\rho\beta)_f + \phi(\rho\beta)_s \\ K_{nf} = K_f\left[\frac{K_s + 2K_f - 2\phi(K_f - K_s)}{K_s + 2K_f + 2\phi(K_f - K_s)}\right] \end{array}\right\} \qquad (6)$$

The radiative heat term by using the Roseland approximation is given by

$$q_r = \frac{-4\sigma^*}{3k^*}\frac{\partial \bar{T}^4}{\partial \bar{z}} \qquad (7)$$

$$\bar{T}^4 \cong 4\bar{T}_\infty^3 \bar{T} - 3\bar{T}_\infty^4 \qquad (8)$$

$$\frac{\partial q_r}{\partial \bar{z}} = -\frac{16\sigma^* \overline{T'}^3_\infty}{3k^*} \tag{9}$$

$$\left.\begin{aligned}
& z = \frac{U_o \bar{z}}{v_f},\ u = \frac{\bar{u}}{U_o},\ v = \frac{\bar{v}}{U_o},\ t = \frac{U_o^2 \bar{t}}{v_f}, \\
& n = \frac{v_f \bar{n}}{U_o^2},\ \bar{w} = -w_0,\ \theta = \frac{\bar{T} - \bar{T}_\infty}{\bar{T}_w - \bar{T}_\infty},\ S = \frac{w_o}{U_o^2}, \\
& K = \frac{\rho_f k U_o^2}{v_f^2},\ M = \frac{\sigma B_o^2 v_f}{\rho_f U_o^2},\ R = \frac{2\Omega v_f}{U_o^2}, \\
& \Pr = \frac{(\rho C_p)_f v_f}{K_f},\ F = \frac{4\sigma^* T_\infty^3}{kk^*}, \\
& Q = \frac{Q_H v_f^2}{K_f U_0^2}
\end{aligned}\right\} \tag{10}$$

Substituting nanofluid properties Eq. (6) and Eqs (7) – (10) in the Eqs (2) – (4) we get the following non-dimensionalequations.

$$(1-\phi)^{2.5}\left[(1-\phi)+\phi\left(\frac{\rho_s}{\rho_f}\right)\right]\left[\frac{\partial u}{\partial t} - S\frac{\partial u}{\partial z}\right] = \frac{\partial^2 u}{\partial z^2} + (1-\phi)^{2.5}\left\{\begin{aligned} &\left[(1-\phi)+\phi\left(\frac{\rho_s}{\rho_f}\right)\right]Rv \\ &+\left[(1-\phi)+\phi\left(\frac{(\rho\beta)_s}{(\rho\beta)_f}\right)\right]\theta \\ &-\left(M+\frac{1}{K}\right)u \end{aligned}\right\} \tag{11}$$

$$(1-\phi)^{2.5}\left[(1-\phi)+\phi\left(\frac{\rho_s}{\rho_f}\right)\right]\left[\frac{\partial v}{\partial t} - S\frac{\partial v}{\partial z}\right] = \frac{\partial^2 v}{\partial z^2} + (1-\phi)^{2.5}\left\{\begin{aligned} &-\left[(1-\phi)+\phi\left(\frac{\rho_s}{\rho_f}\right)\right]Ru \\ &-\left(M+\frac{1}{K}\right)v \end{aligned}\right\} \tag{12}$$

$$\Pr\left[(1-\phi)+\phi\left(\frac{(\rho C_p)_s}{(\rho C_p)_f}\right)\right]\left[\frac{\partial \theta}{\partial t} - S\frac{\partial \theta}{\partial z}\right] = \left[\frac{k_{nf}}{k_f} + \frac{4F}{3}\right]\frac{\partial^2 \theta}{\partial z^2} - Q\theta \tag{13}$$

Subject to initial and boundary conditions in dimensionless form are:

$$\left.\begin{aligned}
& for\ t < 0\ \{\forall z \quad u(z,t) = 0, v(z,t) = 0, \theta(z,t) = 0 \\
& for\ t \ge 0 \begin{cases} at\ z = 0 & u(0,t) = 1 + \frac{\varepsilon}{2}(e^{int} + e^{-int}), \\ & v(0,t) = 0,\ \theta(0,t) = 1 \\ as\ z \to \infty & u(\infty,t) \to 0,\ v(\infty,t) \to 0, \\ \theta(\infty,t) \to 0 & \end{cases}
\end{aligned}\right\} \tag{14}$$

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III. NUMERICAL SOLUTION BY FEM

The transformed system of coupled partial differential Eqs. (11) - (13) under the boundary condition Eq. (14) are solved numerically by using the extensively-validated and robust method known as finite element method. This method has five fundamental steps which are discretization of the domain, derivation of element equations, assembly of element equations, imposition of boundary conditions and solution of the assembled equations. An excellent description of these steps presented in the text book Reddy [9], Bathe [10].

IV. GRID INDEPENDENCE STUDY

The grid independent test is carried out by dividing the entire domain into successively sized grids 61x61, 81x81 and 101x101. For all computations 101 intervals of equal length 0.01 is considered. At each node functions are to be evaluated. Hence after assembly of elements a set of 303 non-linear equations are formed, consequently an iterative scheme namely Thomas algorithm is adopted to solve the equations. A convergence criterion based on relative difference between two successive iterations was used and the procedure is terminated when difference reached to 10^{-7}. The skin friction coefficient and the Nusselt number are defined respectively as:

$$C_f = \frac{\bar{\tau}_w}{\rho_f U_0^2} \quad and \quad Nu = \frac{\bar{x}\bar{q}_w}{k_f(\bar{T}_w - \bar{T}_\infty)}.$$

V. VALIDATION OF THE CODE

To ensure the correctness of this numerical scheme, in the absence of thermal radiation, a comparison of present results for skin friction coefficient and Nusselt number with the results obtained through analytical approach are made. It confirms that present results are in excellent agreement with the results reported by Hamad and Pop[8], which are shown in table 2. Therefore the developed code can be used with the great confidence to study the problem considered in this paper.

Table 2: Comparison of Skin Friction and Nusselt Number for various values of $\Pr$ when $F = 0, K \to \infty$.

	Hamad and Pop (2011)		Present Study	
$\Pr$	C_f	Nu	C_f	Nu
0.5	2.3159708	5.9674	2.3159709	5.9674101
1.0	2.2567503	6.0461	2.2567504	6.0461012
1.5	2.1972895	6.1259	2.1972896	6.1259001
2.0	2.1376083	6.2066	2.1376084	6.2066013

VI. RESULTS AND DISCUSSION

The primary interest of this paper is to investigate numerical study of the effects of Volume fraction ϕ and thermal radiation F on water based nanofluids ($Cu - water$ and $Al_2O_3 - water$) which are saturated in porous medium. Additionally the influence of M and K on the nanofluid velocity, temperature as well as on the local Skin-friction and Nusselt number distribution for two different types of nanofluids is discussed numerically. And also the results are represented graphically in Figs 2 to 8. Furthermore Table1 shows thermo physical properties of base fluid (water), copper (Cu), alumina (Al_2O_3) and (TiO_2).

Table 1: Thermo-physical properties of water and nanoparticles

Physical properties	Water	Cu	Al_2O_3	TiO_2
Cu(j/kg k)	4179	385	765	686.2
ρ(kg/m3)	997.1	8933	3970	4250
K(W/m k)	0.613	400	40	8.9538
$\beta X10^{-5}$(1/k)	21	1.67	0.85	0.9

Water based nanofluid velocity profiles for different values of rotational parameter R is presented in Fig. 2,in which nanofluid velocity profiles are decreasing with the increase of rotational parameter, Finally it can be observed from the graph that (Al_2O_3) has less velocity than the $Cu - water$ nanofluid. Here it is to be pointed that coriolis body forces arise in momentum equations via $-Rv$ and Ru respectively

An application of magnetic field to an electrically conducting fluid produces dragline force (Lorentz force), which acts against the relative motion of the fluid. This force has a tendency to slowdown the motion of the fluid in the boundary layer. This physical behavior is illustrated in Fig. 3. An increase of magnetic field M along the surface causes to decrease the velocity of nanofluid. In addition, it is also observed that $Al_2O_3 - water$ has comparatively less velocity than that of $Cu - water$. Permeability K is the property of porous material which measures the ability for fluids to pass through it. An increase in the permeability of the porous medium leads to the rise in the flow of fluid through it. This physical behavior is illustrated in Fig.4, which shows the effect of permeability parameter K on the nanofluid velocity distribution. As K increases the nanofluid velocity increases. This is due to increase in thermal boundary layer thickness. Therefore porous medium impact is significant on the boundary layer growth. Furthermore this figure depicts that $Al_2O_3 - water$ nanofluid exhibits relatively less velocity than that of the $Cu - water$ nanofluid.

Whenever the volume fraction ϕ of nanoparticles increases, the thermal conductivity of fluid and thickness of thermal boundary layer increases, this physical behavior is exemplified in Figs. 5 and 6. Here in these graphs the velocity profiles are decreased and temperature distribution profiles are increased with the increase of volume fraction parameter of nanofluid respectively. However, the thermal conductivity of water based nanofluids increases as the nanoparticle size increases because the low viscosity of the base fluid promotes the clustering of nanoparticles and these particle clustering forms an interconnecting channels for thermal energy to propagate. So as volume fraction increases the thermal conductivity of water based nanofluid is enhanced. Furthermore graphs are exhibiting that the velocity profiles for $Al_2O_3 - water$ nanofluid is relatively lesser than that of $Cu - water$ nanofluid due to decrease in thickness of the boundary layer, while the temperature distribution in the Cu-water nanofluid is greater than $Al_2O_3 - water$ nanofluid, since by the reason of the fact that copper has significantly high conductivity than the alumina and therefore, the thickness of thermal boundary layer of $Cu - water$ nanofluid is greater than the $Al_2O_3 - water$ nanofluid.

For various values of thermal radiation F , the velocity and temperature of profiles of water based nanofluids ($Cu - water$ and $Al_2O_3 - water$) are shown in the Fig. 7 and 8. It is evident from the figures that with the increase of thermal radiation parameter values, velocity and temperature distribution across the boundary layer increases this is due to the reason that the increase of thermal radiation causes to increase in the thickness of thermal boundary layer. In addition $Cu - water$ has a great velocity and temperature profile than that of $Al_2O_3 - water$ nanofluid.

VII. CONCLUSIONS

The novelty of present work is the dimensionless equations solved numerically by using finite element method. Significant findings from the study are an increase in permeability parameter and thermal radiation tendto enhance the velocity of the nanofluid and it is observed that velocity in the case of Al_2O_3-water is less than that of Cu-waternanofluid. An enhancement of other parameters M ,ϕ and R tend to decelerates the velocity of the nanofluid and also it is noticed that Cu-water is comparatively morevelocity than that of $Al_2O_3 - water$ nanofluid. Thermal radiation and Volume fraction tend to accelerate the temperature profiles.

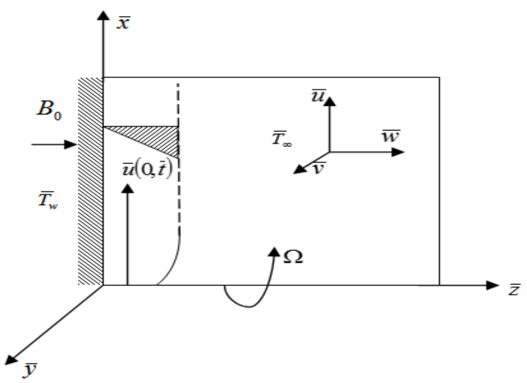

Fig. 1. Physical model and coordinate system

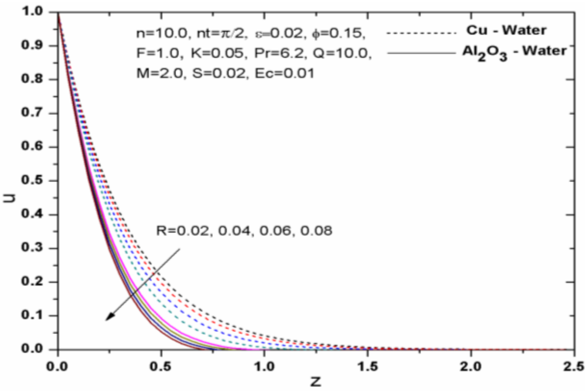

Fig. 2. Velocity Profiles for R

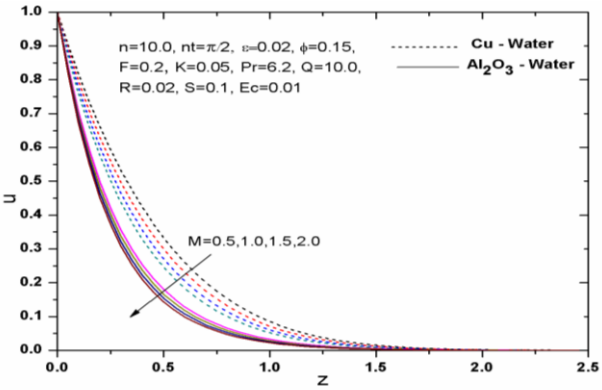

Fig. 3. Velocity Profiles for M

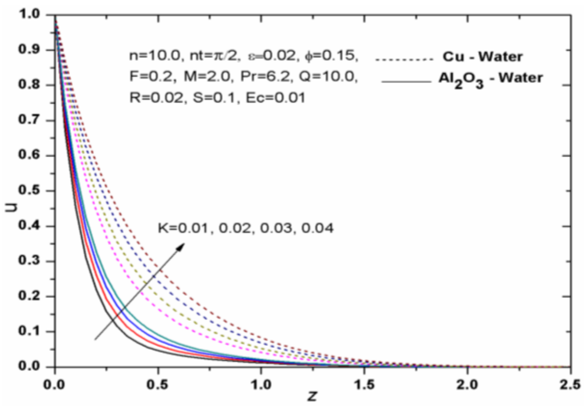

Fig. 4. Velocity profiles for K

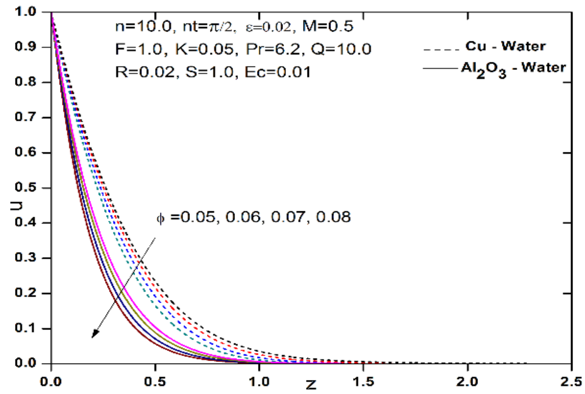

Fig. 5. Velocity Profiles for ϕ

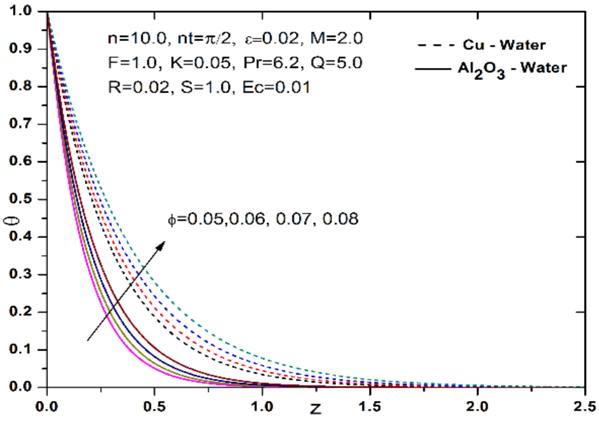

Fig. 6. Temperature Profiles for ϕ

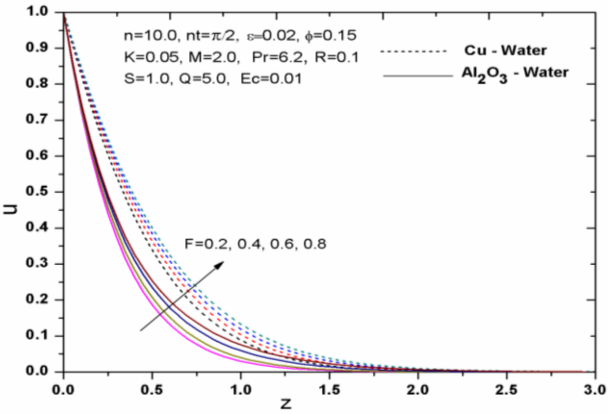

Fig. 7. Velocity Profiles for F

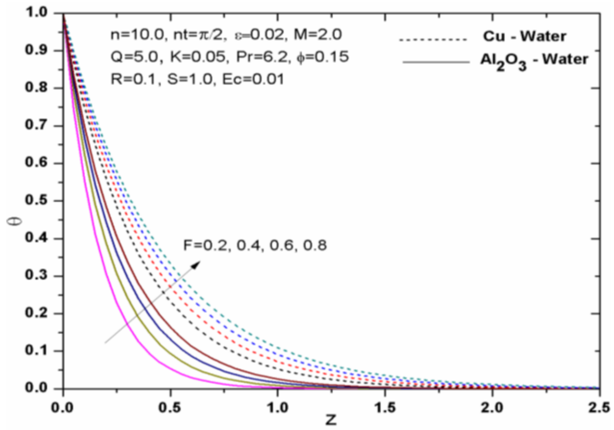

Fig. 8. Temperature profiles for F

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Nomenclature

B_0	Constant applied magnetic field
E	Applied electric field
F	Thermal radiation
g	Acceleration due to gravity
K	Permeability parameter
k	Permeability of porous medium
k^*	Mean absorption coefficient
M	Dimensionless magnetic field parameter
C_f	Skin friction coefficient
Nu	Nusselt number
$\Pr$	Prandtl number
Q	Non-dimensional heat source parameter
Q_H	Dimensional heat source
q_r	Radiative heat flux
q_w	Heat flux from the plate
R	Dimensional rotational parameter
S	Suction parameter
T	Local temperature of the nanofluid
T_w	Wall temperature of the fluid
T_∞	Temperature of the ambient nanofluid
U_0	Characteristic velocity
w_0	Normal velocity
(x, y, z)	Cartesian coordinates
(u, v, w)	Velocity component along x, y, and z axes.

Greek symbols

α	Thermal diffusivity
α_f	Thermal diffusivity of the fluid
α_{nf}	Thermal diffusivity of the nanofluid
β	Thermal expansion coefficient
β_f	Coefficient of thermal expansion of the fluid

β_s	Coefficient of thermal expansion of the solid
ρ_f	Density of the fluid friction
ρ_s	Density of the solid friction
ρ_{nf}	Density of the nanofluid
v	Kinematic viscosity
v_f	Kinematic viscosity of the fluid
μ	Dynamic viscosity
μ_{nf}	Viscosity of the nanofluid
σ	Electrical conductivity of the fluid
σ^*	Stefan–Boltzmann constant
$(\rho C_p)_{nf}$	Heat capacitance of the nanofluid
ε	Small constant quantity
θ	Non-dimensional temperature

Subscripts

f	Fluid
s	Solid
nf	Nanofluid
w	Condition at the wall
∞	Condition at free stream

BIOGRAPHIES

Thirupathi Thumma is as an Assistant Professor of Mathematics in the Department of Mathematics, B V Raju Institute of Technology. Medak, India. He did M.Sc. in Mathematics from Kakatiya University, Warangal in 2006, and received M. Tech [CSE] from JNTUH in 2008. Presently He is pursuing Ph.D. from GITAM University. He has 10 years of teaching experience in various colleges at different levels. His research interests include, fluid flow in porous media, heatand mass transfer, fluid dynamics, magneto hydrodynamics utilizing nanofluids and micropolar fluids. He has published five research papers in internationally reputed journals in Mathematics and Computer Science.

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