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Exponentially Weighted Methods for Forecasting Intraday Time Series with Multiple Seasonal Cycles

S.C. Thasleema[1], B. Sarojamma[2]

Research Scholar, Department of Statistics, S.V. University, Tirupati, A.P, India[1]

Assistant Professor, Department of Statistics, S.V. University, Tirupati A.P, India[2]

Abstract: Intraday data plays a vital role for Atmospheric Sciences for Wind Speed, Wind Wave length, Temperature etc. hourly data.In this paper, we have introduced two new Intraday data models i.e. New Exponential Smoothing and Trignometric model for intraday data. These two models are deduced according to seasons i.e. Summer, Winter and Rainy seasons. In this paper, two measures of accuracy are used. They are Mean Square error and Root Mean Square Error (RMSE). These two models are empirically tested using Atmospheric data of Gadanki India.

Keywords: New Exponential Smoothing, Intraday data, MSE, RMSE.

I. INTRODUCTION

Time Series data is many types. Some of them are intraday data, intra week data, interval based data etc. These type of datas are used mainly for weather forecasting, trading in Sensex, etc. James W. Taylor introduced many articles on intraday data. James W. Taylor in his article exponentially weighted methods for intraday time series with multiple seasonal cycles, enlighten five new univariate exponentially weighted methods for forecasting intraday time series and that consists of both intraweek and intraday seasonal cycles.

The five methods introduced are Holt-Winter Taylor exponential smoothing model. Intraday cycle (IC) exponential smoothing, double seasonal total and split exponential smoothing, discounted weight regression with trigonometric terms. Splines using Regression and DWR and spline based exponential smoothing.

A comparision of Univariate Time Series methods for Forecasting Intraday Arrivals at a Call Center was given by James W. Taylor in his paper. He analyzed five series of intraday arrivals for call centers operated by a retail bank in the UK. He developed number of models and are ARIMA modeling, periodic AR modeling. Extension of Holt-Winters exponential smoothing for the case of two seasonal cycles, robust exponential smoothing based on exponentially weighted least absolute deviations regression and dynamic harmonic regression.
Discounted weighted estimation was given by Ameen & Harrison. Detecting intraday periodicities with application to high frequency exchange rates given by Brooks & Hinich. Forecasting hourly electricity demand using time-varying Splines given by Harvey & Koopman.
Lam et al published a model on Short-term hourly traffic forecasts using Hong Kong annual traffic census. Piecewise regression using cubic Splines was given by Poirier, Dynamic harmonic regression was given by Young et al.

II. METHODOLOGY

If the data is intraday data i.e., with in day data may be in hours or 24 observations etc. and in our data 24 hours data for 7 years. Now we are fitting two models for data, i.e.

1. New Exponential Smoothing model for intraday data.
2. Trigonometric model for intraday data.

1. New Exponential smoothing model for intraday data.

The year contains three seasons i.e., summer, winter and rainy seasons. Summer season contains March, April, May and June months, winter season contains January, February, November and December months and rainy season contains July, August, September and October months. We write new exponential smoothing equation for intraday data is,

$$Y_T = L_T + W_T + S_T + R_T + \varepsilon_T.$$

Where,
Y_T = time series variables at time "T".
W_T = winter season, S_T = summer season and R_T = rainy season respectively at time 'T'. ε_T is error.

$$L_T = L_{T-1} + \alpha\varepsilon_T$$

$W_T = W_{T-1}\beta$ hour value / day value; for each hour in a day of January, February, November and December.
$R_T = R_{T-1}\gamma$ hour value / day value; for each hour in a day of July, August, September and October.
$S_T = S_{T-1}\delta$ hour value / day value; for each hour in a day of March, April, May and June.
For computation of β, γ and δ, we compute seasonal wise errors and absolute errors.

β = summer / summer absolute error.
γ = winter / winter absolute error.
δ= rainy error / rainy absolute error.

2. Trigonometric model for intraday data.

Trigonometric model fitted for day data for 24 hours observations. We defined sine wave by assuming 50 % of observations in increase mode and 50% of observations in decreasing mode.

$$Y_t = a_1 + a_2 \sin(2\pi Dt / 24) + a_3\cos(2\pi Dt / 24)$$

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Equivalently, this equation can be written as

$Y_t = a_1 + a_2 \sin(2\pi Dt / 24) + a_3 \cos(2\pi Dt/24)$

$Y_t = a_1 f_1(t) + a_2 f_2(t) + a_3 f_3(t)$.

Where,

$f_1(t) = 1$, $f_2(t) = \sin(2\pi Dt/24)$, $f_3(t) = \cos(2\pi Dt/24)$

a_1, a_2 and a_3 coefficients are estimated using ordinary least square method.

Separate trigonometric models are fitted for rainy, summer and winter seasons.

Another trigonometric model fitted for data by assuming 6 hours data is increasing and another 6 hours data decreasing and same process for another 12 hours. For these we fit trigonometric model in another form as

$Y_t = a_1 + a_2 \sin(2\pi Dt/12) + a_3 \cos(2\pi Dt/12)$

Equivalently, this equation can be written as

$u_1 = a_1 f_1(t) + a_2 f_2(t) + a_3 f_3(t)$

Where

$f_1(t) = 1$, $f_2(t) = \sin(2\pi Dt/12)$, $f_3(t) = \cos(2\pi Dt/12)$

Constants a_1, a_2 and a_3 are estimated using ordinary least square estimates.

Trigonometric model is fitted separately for mid night to 12:00 Noon and 12:00 Noon to Mid night for rainy, winter and summer seasons.

MEASURES OF ACCURACY

In this paper we used two measures of accuracy

1. Mean Square Error (MSE) and
2. Root Mean Square Error (RMSE)

1. Mean Square Error (MSE)

Difference between original value and absolute value gives an error. Mean of squared error gives MSE.

2. Root Mean Square Error (RMSE)

Positive square root for mean square error gives RMSE. The model which posses less MSE or RMSE is the best model compared with another model.

III. EMPIRICAL INVESTIGATIONS

For fitting of new exponential smoothing model for intraday data is

$$Y_T = L_T + W_T + S_T + R_T + \varepsilon_T$$

Where,

Y_T is time series variables at time 'T'.

L_T, W_T, S_T and R_T are level, winter, summer and rainy seasons are at time 'T'.

ε_T is error at time 'T'.

Computed error and absolute errors for three seasons of data for 2008 to 2010 are as follows.

For computation of β, γ, and δ, we compute seasonal wise errors and absolute error.

TABLE - I

Seasonal wise		2008	2009	2010
Summer	Error (E)	97244	-85224	-43337
	Absolute Error (AE)	36944	375626	417147
	E / AE	0.26322	-0.22689	-0.10123
Winter	Error (E)	-272481	121847	-268896
	Absolute Error (AE)	375871	292873	354730
	E / AE	-0.72493	0.41604	-0.75803
Rainy	Error (E)	112473	-17796.5	188845
	Absolute Error (AE)	403227	220227.5	462971
	E / AE	0.278932	-0.08081	0.4079

We fit new exponential smoothing model for intraday data is by base year 2007 to 2008 year, 2008 to 2009 and 2009 to 2010.

$$Y_T = L_T + W_T + S_T + R_T$$

Where, $L_T = L_{T-1} + \alpha\varepsilon_T$

$W_T = W_{T-1}\,\beta$ hour value / day value

$R_T = R_{T-1}\gamma$ hour value / day value

$S_T = S_{T-1}\delta$ hour value / day value

By taking the above calculations as test set and we proceed to fit same model to 2011 and 2012 data, computation Error (E), Absolute Error (AE) and E/AE are shown in table - II.

TABLE – II

Year	Season	Error (E)	Absolute Error (AE)	E / AE
2011	Summer	20791	259491	0.080122
	Winter	51400	362776	0.1416
	Rainy	10267.5	252298.5	0.0407
2012	Summer	24062	245750	0.0979
	Winter	63077	240437	0.2623
	Rainy	120877	334639	0.3612

Root mean Square error of mode i.e., New exponential smoothing model for intraday data is

TABLE - III

	RMSE		
YEAR	RAINY	SUMMER	WINTER
2011	101.51	95.33	168.81
2012	101.71	93.56	178.05

Trigonometric model for intraday data i.e., by assuming 24 hours a day model is

$$Y_t = a_1 + a_2 \sin(2\pi Dt/24) + a_3 \cos(2\pi Dt/24)$$

Constants a_1, a_2 and a_3 are estimated by using ordinary least square estimates separately for rainy, summer and winter seasons.

Rainy $Y_t = 11.535 + 0.423 \sin(2\pi Dt/24) + 1.060 \cos(2\pi Dt/24)$

Summer $Y_t = 10.472 - 0.999 \sin(2\pi Dt/24) + 1.242 \cos(2\pi Dt/24)$

Winter $Y_t = 11.442 - 0.160 \sin(2\pi Dt/24) + 0.341 \cos(2\pi Dt/24)$

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Mean square error and root mean square error for models are

TABLE - IV

YEAR	RAINY	SUMMER	WINTER
MSE	11.5001	11.5056	11.4998
RMSE	3.3912	3.3920	3.3911

If by splitting day into two parts i.e., from midnight to 12:00 noon is one group and from 12:00 noon to midnight is another group and we fit two separate trigonometric models of two parts. General trigonometric model fitted for data is

$$Y_t = a_1 + a_2 \sin(2\pi Dt/12) + a_3 \cos(2\pi Dt/12)$$

Trigonometric model is modified into the form

$$Y_t = a_1 f_1(x) + a_2 f_2(x) + a_3 f_3(x)$$

Where, $f_1(x) = 1$

$f_2(x) = \sin 2\pi Dt / 12$

$f_3(x) = \cos 2\pi Dt / 12$

Models fitted for rainy, summer and winter seasons separately are

Rainy $Y_t = 6.072 + 1.16 \sin(2\pi Dt/12) - 0.459 \cos(2\pi Dt/12)$ 1 to 12

$= 17.979 - 0.157 \sin(2\pi Dt/12) + 0.178 \cos(2\pi Dt/12)$ 13 to 24

Summer $Y_t = 6.968 + 1.95 \sin(2\pi Dt/12) - 0.758 \cos(2\pi Dt/12)$ 1 to 12

$= 16.995 + 1.117 \sin(2\pi Dt/12) + 2.217 \cos(2\pi Dt/12)$ 13 to 24

Winter $Y_t = 6.255 + 0.215 \sin(2\pi Dt/12) - 1.944 \cos(2\pi Dt/12)$ 1 to 12

$= 17.657 - 0.847 \sin(2\pi Dt/12) + 1.219 \cos(2\pi Dt/12)$ 13 to 24

Mean square error and root mean square error for rainy, summer and winter with two time periods are as follows

TABLE -V

	YEAR	RAINY	SUMMER	WINTER
MSE	1 to 12	6.0004	5.9978	5.9988
	13 to 24	18.0002	18.0079	18.0020
RMSE	1 to 12	2.4496	2.4490	2.4492
	13 to 24	4.2427	4.2436	2.2429

IV. SUMMARY AND CONCLUSIONS

We fitted new exponential smoothing model for intraday data as

$$Y_T = L_T + W_T + S_T + R_T + \varepsilon_T$$

Where L_T, W_T, S_T, R_T & ε_T are level, Winter season, summer season, rainy season and error at time 'T'.

$L_T = L_{T-1} + \alpha\varepsilon_T$

$W_T = W_{T-1}\beta$ hour value / day value.

$R_T = R_{T-1}\gamma$ hour value / day value

$S_T = S_{T-1}\delta$ hour value / day value

Where β, γ and δ are estimated using ratio of error to absolute error with their respective seasons.

Root mean square error for the model to years 2011 and 2012.

TABLE-VI

YEAR	RMSE		
	RAINY	SUMMER	WINTER
2011	101.51	95.33	168.81
2012	101.71	93.56	178.05

Trigonometric models are fitted to data by taking 24 hours a day and 12 hours a day models as follows.

24 hour trigonometric model is

$$Y_T = a_1 + a_2 \sin(2\pi t/24) + a_3 \cos(2\pi t/24)$$

12 hour trigonometric model is

$$Y_T = a_1 + a_2 \sin(2\pi t/24) + a_3 \cos(2\pi t/24)$$

Constants a_1, a_2 and a_3 are estimated using ordinary least squares method.

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BIOGRAPHIES

Mrs. S.C. Thasleema She is presently pursuing Ph.D. in the Department of Statistics, Sri Venkateswara University, Tirupati, Andhra Pradesh, India. Her topic is "Some models for Time series analysis". She has published five papers in National and International Journals. She has presented five papers in International Conference and two in National Conference. She is a Life Member of International Indian Statistical Association (IISA).

Dr. B. Sarojamma received Ph.D. degree in Statistics in 2006 from Sri Venkateswara University, Tirupati, Andhra

Pradesh, India. She taught the subjects Time series and Forecasting, Operations Research for Management, Advanced Operations Research, Distribution Theory, Sampling Techniques and Biostatistics, etc. to the students of M.Sc. Statistics, Applied Statistics, Statistics with computer applications, M.Sc Home science, M.Sc Aquaculture, Bio-technology and Bio-informatics. She has given several

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Invited lectures in national/International workshops/Conferences. She has published more than 45 research papers in internationally reputed journals and presented more than 60 papers at national/International Conferences/Symposia. She is Life member (LM) in many Professional Bodies like Indian Society for Probability and Statistics (ISPS), Andhra Pradesh Society for Mathematical Sciences Hyderabad, International Indian Statistics Association (IISA), Cochin, Indian Mathematical Society (IMS), Aurangabad and Indian Statistical Association (ISA), Pune. At present she is working as an Assistant Professor in the Department of Statistics, S. V. University, Tirupati.

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