

Analysis and Verification of Resistance Co-Efficient with Different Flow Parameters Having Different Bed Conditions to Open Channel Flow

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Abstract: The resistance to open channel flow has long been an important aspect of work of the hydraulic engineers. The earliest means of quantification of Manning's n and Chezy's C in open channel flow, Darcy-Weisback's friction factor f in pipe flow have been still in use. The use of these resistance coefficients (n , C and f) in the analysis of flow in open channel and pipe is normally selected on the basis of bed roughness for open channel and inside diameter roughness of pipe. The flow is assumed to be fully developed i.e. turbulent flow. Different researchers claim that values of these coefficients are very much dependent on states of flow. In this work, how these resistance coefficients change with changing bed roughness and flow conditions are shown with the help of experimental works. The results show that use of constant values of n or f based on roughness is not recommended.

Keywords: Manning's n , Chezy's C , friction factor, Reynolds no. R_e , roughness, open channel flow.

I. INTRODUCTION

The modern concept of using resistance equation is to replace the Manning's n or the Chezy's C , by the friction factor f with the adaptation of the equivalent diameter of an open channel and pipe, whatever may be the resistance law. The proper assessment of resistance co-efficient in changing flow situation is a vital factor in computation of free surface profile, design of canal, flood control, training of river channel, computation of water hammer in hydro-electric supply pipes etc. Steady and unsteady surface profiles are very much sensitive to different resistance co-efficient with various flow conditions i.e. laminar to turbulent zone of flow. The present study is mainly concerned with the analysis and verification of resistance co-efficient in open channel flow with different flow parameters having different surface roughness.

II. REVIEW OF RESISTANCE FORMULAE

A brief review of the resistance formulae of open channel and effects of roughnesses to the flow resistance are presented here. The empirical formulae of Manning's and Chezy's have been chosen for analysis. Modern concept of using Darcy-Weisback's friction factor f in open channel and pipe has been favoured for analysis as the resistance co-efficient. Many researchers have studied effects of natural and artificial roughnesses in open channel flow resistance for various bed conditions are outlined here.

As early as 1768 the French Engineer Antonie Chezy was adopting probably the first uniform flow formulae, the famous Chezy formula. He reasoned that the resistance would vary with the wetted perimeter and with the square of the velocity, and that the force to balance this resistance

would vary with the area of the cross-section and with the slope. Therefore $V^2P/(AS)$ or $V^2/(RS)$ would be constant for any one channel, and would be the same for any similar channel.

Mathematically it is usually expressed as

$$V = C\sqrt{RS} \quad (1)$$

where V is the mean velocity in m/sec, R is the hydraulic radius in m, S is the bed slope in uniform flow and C is the factor of flow resistance called Chezy's C . The first systematic and extensive effort to discover how this co-efficient varied under different conditions was begun by Darcy in 1855 and continued after his death by H.E. Banzin (1865). The wall surface included cement, brick, fine gravel, coarse gravel, plank, and wood strips transverse to the flow. Rectangular, trapezoidal, triangular, and semicircular channel shapes were tested. The results showed that, while C dependent principally on the roughness of the wall and for any given roughness it was less for large channels than for small channels. Banzin (1865) proposed a formula that is equivalent to $1/C^2 = a+b/R$, in which 'a' and 'b' depends on the wall roughness.

Ganguillet and Kutter (1869) published a formula expressing the value of 'C' in terms of the slope 'S', hydraulic radius 'R', and the co-efficient of roughness 'N'. In metric unit the formula is

$$C = \frac{23 + \frac{0.0155}{s} + \frac{1}{n}}{1 + (23 + \frac{0.0155}{s}) \frac{n}{\sqrt{R}}} \quad (2)$$

The co-efficient n in this formula is specially known as Kutter's n .

In 1891, Robert Manning presented a formula which is later modified to its present well-known form in metric unit is

$$V = \frac{1}{n} R^{2/3} S^{1/2} \quad (3)$$

Here, V is the mean velocity in meter/sec, R is the hydraulic radius in m, S is the bed slope in uniform flow, and n is the co-efficient of roughness, specially known as Manning's n . This formula was developed from seven different formulas, based on Banzin's experimental data, and further verified by 170 observations. Owing to its simplicity of form and to the satisfactory results it tends to wide practical applications, and thus the Manning formula has become the most widely used of all uniform flow formulae for open channel flow computation.

Comparing the Chezy's formula with Manning formula, it can be seen that –

$$C = \frac{1}{n} R^{1/6} \quad (4)$$

Friction factor for open channel flow can be written as –

$$f = \frac{8gRS}{v^2} \quad (5)$$

The following relationship hold between Chezy's C and Manning's n and friction factor f .

$$C = \sqrt{\frac{8g}{f}} \quad (6)$$

From the equation (5) and (6) it can be shown that –

$$n = \sqrt{\frac{f}{8g}} R^{1/6} \quad (7)$$

These were all based on empirical study. Modern concept of resistance to flow was proposed by Leonard Prandtl (1904) based on boundary layer theory which has profoundly affected the resistance to flow in open channels as well as pipes.

In 1923, L. Hopf and K. Forman published the first paper on the measurement of hydraulic roughness in which modern concepts were considered. Hopf showed that, in general friction factor ' f ' is a function of the relative roughness, the Reynolds number, and the shape of cross-section. He concluded that the wall effect on ' f ' is due to two separate causes- the waviness of the wall and its roughness.

Prandtl (1904) had deduced a formula for ' f ' as a function of the Reynolds number for smooth pipes. By the next year, J. Nikuradse (1950) had demonstrated that, for pipes of uniform diameter, if the walls were rough and Reynolds number sufficiently high, ' f ' become dependent on the relative roughness only. For lesser roughness or Reynolds number, he found that ' f ' increased with Reynolds number. He developed a formula for ' f ' in terms of the ratio of the radius of the pipe to the diameter of the sand grains that he had used to roughen the inside of the pipe. Nikuradse's data have served as the basis for many subsequent analysis of frictional resistance in pipes and open channels for smooth, partly rough, and fully rough flow. However, Nikuradse used uniform sand grains for roughness and this produced an increase in ' f ' with

Reynolds number in partly rough flow. C. F. Colebrook (1937) investigated this same region using non-uniform roughness. It was found that ' f ' decreased somewhat in this region and, Colebrook and White (1938) gave the familiar transition formula for pipes, which can be converted to:

$$\frac{1}{\sqrt{f}} = -2 \log \left(\frac{K_s}{14.83 R} + \frac{2.51}{Re\sqrt{f}} \right) \quad (8)$$

Equation (8) is known as Colebrook-White equation in which K_s is the sand grain. R is the hydraulic radius, Re is the Reynolds no. For the purpose of further analysis, this formula will be written in the general form:

$$\frac{1}{\sqrt{f}} = -c \log \left(\frac{K_s}{aR} + \frac{b}{Re\sqrt{f}} \right) \quad (9)$$

A.D. Altshul (1952) presented a paper primarily on pipes but also including open channels. As a substitute for Colebrook's formula, he recommended:

$$\frac{1}{\sqrt{f}} = 1.8 \log \left[\frac{Re}{Re_a + 7} \right] \quad (10)$$

He recommended 1.8 as a co-efficient instead of 2.0 because he claimed that it fitted the data better, he stated that the co-efficient may vary between 1.2 and 2.8 for different types of roughness and 2.0 only in the case of the uniform roughness. A new concept of roughness was advanced by Morris (1955). He assumed that the loss of energy in turbulent flow over a rough surface is largely due to the formation of waves behind each roughness element. Under this concept, the longitudinal spacing of the roughness elements is the roughness paramount importance in rough turbulent flow.

The state of behaviour of open channel flow is governed basically by the effects of viscosity and gravity relative to the inertial forces of the flow. The surface tension of water may affect the behaviour of flow under certain circumstances, but it does not play a significant role in most open channel problems encountered in engineering.

The flow is laminar if the viscous forces that viscosity plays a significant part in determining flow behaviours. In laminar flow, the water particles appear to move in definite smooth paths, or streamlines, and infinitesimally thin layers of fluid seem to slide over adjacent layers.

The flow is turbulent if the viscous forces are weak relative to the inertial forces. In turbulent flow, the water particles move in irregular paths which are neither smooth nor fixed but which in the aggregate still represent the forward motion of the entire stream. Between the laminar and turbulent states there is a mixed or transitional state. The effect of viscosity relative to inertia can be represented by the Reynolds number

The effect of gravity upon the state of flow is represented by a ratio of inertial forces to gravity forces. This ratio is called by the Froude number. It is believed that Froude number may have a definite effect upon the flow resistances in channels at the turbulent flow range. The experimental studies by Jegorow (1940) and Iwagaki (1953) for smooth rectangular channels and by Homma (1952) for rough channels have shown that in the supercritical-turbulent regime of flow, the friction factor is

likely to increase with increasing Froude number. Generally, the effect of gravity is practically negligible where the Froude number is small, say less than 3. A further investigation by Iwagaki indicates that, with increasing Froude number, the friction factor of turbulent flow in both smooth and rough open channels become larger than that in pipes.

From the above review of the resistance formulae of open channel, it has been found that different authors have presented different resistance formulae and equations from time to time. But none of the formulae have been found to be satisfactory. Only Manning's formula, Chezy's formula and Darcy-Weisbach formula have been most suitable for studies of different problems relating to resistance of open channel and pipe flow.

Hence in this study, stress has been given to find out the various relationship between Manning's 'n', Chezy's 'C' and Darcy-Weisbach friction factor 'f' based on experimental data of authors produced in Hydraulics laboratory of Assam Engineering College.

Ben Chie Yen (2002) extended the study of Rouse (1965) by discussing the differences between momentum and energy resistances, between points, cross-sectional and reach resistance coefficients, as well as compound/composite channel resistance. Certain resistance phenomena can be explained with the inner and outer laws of boundary layer theory. The issue of linear-separation approach versus nonlinear approach to alluvial channel resistances was also discussed. This review indicates the need for extensive further research.

Zidan, A.R.A. (2015) demonstrated the application of alternative equations of resistance, such as the rough turbulent formula, the Williamson equation and the Colebrook White equation. Differences between, and limitations of each formula are also presented by him. An approach to the solution of Colebrook White formula in an explicit form in open channels is given and a comparative study between this formula and other explicit formulae is also presented.

A. Few Earlier Experimental Works

Banzin (1865) carried out extensive experiments in order to study the behaviour of resistance co-efficient. He had adopted to carry out his experiment in a channel of 2m wide, and nearly 600m long for a different roughness conditions.

Powell (1944) carried out his study through some experimental works in the laboratory of the institute of Hydraulic Research at Iowa City, Iowa. A total of some two hundred runs on a rectangular channel with eleven roughnesses and forty four slopes were conducted. A new concept of roughness by Morris (1955) was that the loss of energy in turbulent flow over a rough surface is largely due to the formation of wax behind each roughness element. Sayre and Albertson (1963) performed experiment on a rectangular flume 8ft wide, 8 inches depth and 72ft long sheet metal baffles measuring 6inches wide and $1\frac{1}{2}$ inches high were used as roughness element.

Moeller-Hartman (1957) conducted test on triangular flumes with different angles for both rough and smooth surfaces. Tracy and Lester (1961) performed test in a smooth rectangular channel with Froude number varying from 0.14 to 3.96. Biery and Delleur (1961) also produced experimental data on a rectangular flume of 5ft wide, 2ft depth and 64ft long with two different roughnesses. Reinius (1961) carried out experiment on steady uniform flow in open channel with 1.98ft wide and 51.8ft long. The walls of the channel used were smooth bottom steel sheet and glass, bottom of steel balls having a diameter of 4.76mm and bottom of steel balls having a diameter of 9.52mm.

Rajarathan (1976) carried out experiments on a Plexiglas channel of nine inches wide, eight inches deep and thirty two ft. long with four types of roughnesses.

Elaborate experimental works on open channel were conducted by Vanoni (1957), Richard (1966), Brooks (1955) and D.I.H Bar and M.M Das (1986).

The experimental works mentioned above were conducted to generate data to satisfy their own line of investigation. The authors also carried out experimental works in the laboratory to generate data to study the flow resistance mainly in the open channel.

III. EXPERIMENTAL STUDY IN RECTANGULAR GLASS CHANNEL

The experiments were performed in the Hydraulics laboratory of Civil Engineering Department of Assam Engineering College, Guwahati. The experiments were conducted in rectangular open channels which are made of glass. The channel is associated with a water supply system and an iron tank calibrated in litres for measurement of discharge and a movable pointer gauge for measurement of flow depths in the channel. The test section of the channel which was made of glass was 0.102m wide by 0.200 m deep and 4.00 m long. Upstream from the test section was an entrance section of 1.80 m long in which an attempt was made to establish the uniform velocity distribution before the flow entered the test section. To damp down the turbulence of water before entering the test section baffle of wood with small holes have been provided. Despite this provision, the flow was not damped up to expectation and to achieve this wire mesh was fastened to the baffle to reduce the holes. Also in this 1.80 m section a gradual transition of cross-section had been provided to develop uniform flow condition. An adjustable jack was used to set the channel at the different desired bed slope.

The glass channel is attached with a venturi meter and a re-circulated pumping system which is electrically operated.

To begin the experiment, the channel was set to desired slope. Normally for each slope of the channel, at least four runs were made by varying the discharge of the pumps. For each run normal depth was measured with the movable point gauge after the flow become uniform.

The discharge Q was calculated from the following Venturi meter equation.

$$Q = C_d \frac{A_1 A_2}{\sqrt{A_1^2 - A_2^2}} \times \sqrt{2gh} \quad (11)$$

Discharge in the channel was measured by a calibrated iron tank. Velocity and hydraulic mean radius were calculated from the continuity equations.

IV. THEORETICAL EQUATION SELECTED FOR VERIFICATION

Barr and Das (1986) presented direct solution of normal depth with Manning equations for rectangular channel. This explicit equation saves tremendous computing time of implicit solution of normal depth. The following is equation in non-dimensional form.

$$\left(\frac{Y_n}{B}\right) = \left(\frac{Q_n}{B^{8/3} S^{1/2}}\right)^{3/5} \left[1 + 0.855 \left(\frac{Q_n}{B^{8/3} S^{1/2}}\right)^{3/5}\right] \quad (12)$$

Three different types of bed roughness are used in the experiment. More than one hundred and fifty data are produced for 3 different bed roughnesses. Values of normal depths, C, n and f are calculated for various Reynolds numbers. All calculations and plotting are done by computer in Met Lab. The plots are shown in Figure 1 to Figure 3 in three different roughnesses' respectively.

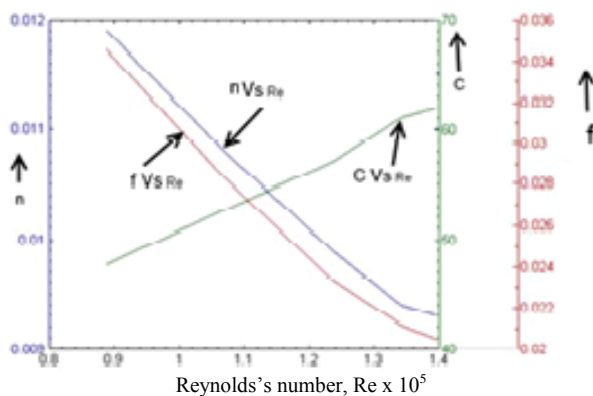


Fig.1 Manning's n, Chezy's C and friction factor f for roughness 1

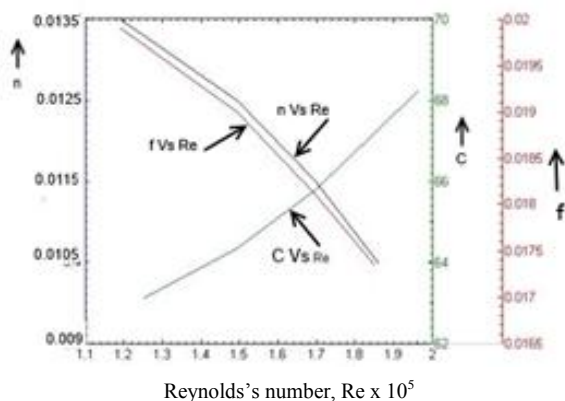


Fig.2 Manning's n, Chezy's C and friction factor f for roughness 2

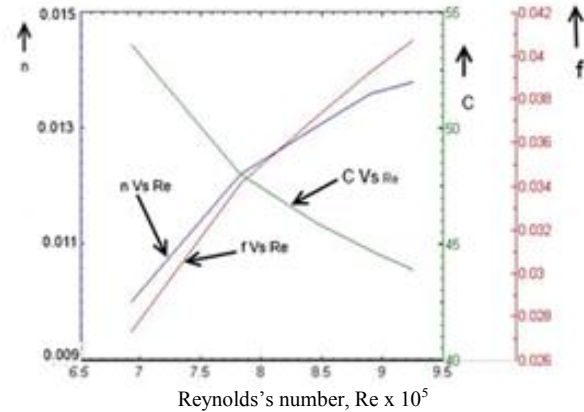


Fig.3 Manning's n, Chezy's C and friction factor f for roughness 3

V. DISCUSSION

Figures 1, 2 and 3 have shown the graphical representations of Manning's n, Chezy's C and friction factor f against Reynolds number Re_e . It has become clear from the figures that 'f' and 'n' always maintain a similar trend of decreasing value as Reynolds number increases whereas C curves is quite opposite in nature. At low roughness with the rapid decrease in Reynolds number 'f', 'n' tends to increase gradually at a very slow rate. But at higher roughness with rapid decrease in Reynolds number 'f', 'n' tends to increase rapidly almost linearly. Chezy's 'C' increases almost linearly with the gradual increase of Reynolds number. However, at rapid increase in Reynolds number 'C' tends to remain constant. Increase of 'C' with the increase of Reynolds number also depends on roughness conditions of the channel bed. At low roughness 'C' increase rapidly with the increase of Reynolds's number.

VI. CONCLUSION

The increasing trend of values of Manning's 'n' with the decrease of depth shows the increase of resistance of flow. This analysis proves that use of constant friction factor, Manning n or Chezy's C is not a good practice that may involve error in design. Resistance coefficient is very much dependent on Reynolds number of flow. Therefore, to have very accurate results, the authors believe that calibrated value of 'n' or C or f is essential as the Manning's equation is still widely used by engineers. Thus a conclusion can be made that the use of constant 'n' and 'C' and 'f' does not produce the very accurate results in assessing resistance to flow in different flow depths. The experimental data of Richard, Venoni and Brooks, Sayre and Albertson, Powell and Posey have shown similar response. So the use of constant resistance coefficient in study of flow in channel or pipe is not recommended.

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