

A Hybrid Simulated Annealing Based Approach to Resolve TOC Product Mix Decision Problem

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Abstract: In recent years, theory of constraints (TOC) has emerged as an effective management philosophy for solving the product mix decision problem with the aim of profit maximization by considering the bottleneck in traditional as well as modern manufacturing plants. One of the key components of TOC application is to enumerate quantity of the various products to be produced keeping in view the system constraints and this is termed as TOC product mix decision problem. It is well known computationally complex problem and thus warrants the application of heuristics technique or AI based optimization tools get to achieve near optimal solution in real time. In this research a hybrid algorithm named tabu-simulated annealing is proposed. The proposed algorithm exploits the beauty of tabu search and SA and ensures the convergence at faster rate.

Keyword: Product mix, perturbation, tabu-list, cooling rate, and aspiration

I. INTRODUCTION

Today's capricious market, manufacturing industries are aiming to increase their throughput by improving their manufacturing and business processes. System constraints present at real shop floor environment prohibit their ability to achieve its goals. These Constraints may be physical / tangible (e.g. machines, vendors etc.) or non-physical / intangible (e.g. policies, performance, measures etc). Develop country like US, Japan, are practicing several management philosophies such as Just in time (JIT), Lean Manufacturing, TQM and Theory of Constraints (TOC) etc. to manage these constraints and increase their productivity. Among them TOC is most prominent overall management philosophy develop by Goldratt [1]. Fox and Goldratt [2] define three important performance indicators, which are used in evaluating of manufacturing case flow. These performance parameters are throughput, inventory, and Operational expenses. If any of the aforementioned performance indicator changes, it will effect financial measurement at strategic level. Thus, TOC philosophy focuses on the goal of manufacturing i.e. to increase throughput with reduction in inventory and operational cost. Key idea of TOC is to determine the system constraints and effectively manage it. When constraints can be easily identified, the five steps process of on-going improvement provides the steps necessary to deal with the constraints. On the other side when constraints are not easily identified, then thinking process provides the tools necessary to identify the core problem or core conflict and provide tools needed to deal with these conflicts effectively.

To manage the constraints, Goldratt [3] proposes five focusing step process of an on going improvement known as five focusing steps or 5FS. These steps are:

1. **Identify** the system's constraint
2. Decide how to **exploit** the system's constraints.
3. **Subordinate** every thing else to the above decisions.
4. **Elevate** the system's constraints.

5. Don't allow **inertia** to become the systems constraint. When a constraint is broken, go back to step one.

Most researchers either increased the number of constrained resources and /or added additional products in Goldratt's[1] product mix problem. Plenert [5] developed an example with multiple constrained resources where TOC product mix algorithm did not provide an optimal or even feasible solution. The failure of TOC is to identify the optimal solution under the existence of multiple constrained resources prompted authors to refine the TOC algorithm to overcome this deficiency. In this research, a TOC product mix problem has been considered and its solution is approached by developing a hybrid algorithm known as tabu-simulated annealing algorithm. \

II. PROBLEM DESCRIPTION

In presence of multiple constraints a product mix decision inherent in the traditional TOC heuristic is described in this section. Main objective function conceptualized here is related to maximization of throughput of several product types. Wear as constraints are related to capacity limit of resources and maximum bound for the quantity of product pertaining to each product.

Mathematical formulation of the problem is described here along with notations.

t_m : throughput of the product type m

q_m : decision variable representing the quantity of the product type m

x_m : processing time required for resource n to produce a type m product

l_n : capacity limit of resource n

d_m : The bound for q_m (market demand for product type m)

i : number of resources

j : number of product mix type

The formulation is given as:

$$\text{Maximize } z = \sum_{m=1}^j t_m q_m \quad \dots(1)$$

Subject to

$$\sum_{m=1}^i x_{mn} q_m \leq l_n \quad n = 1, 2, \dots, i \quad \dots(2)$$

$$0 \leq q_m \leq d_m, \quad m = 1, 2, \dots, j \quad \dots(3)$$

To illustrate the formulation and also to develop its solution a well known problem addressed by Onwubolu [6] has been adopted and given in table 1. Priority analysis data is provided in table 2.

Table1. Capacity analysis and over loads

Resource	Product					Load	Capacity	Utilization	Over-Load
	A	B	C	D	E				
Demand	20	30	40	30	60				
10	2.5	5.5	3.5	2.0	10.0	1015	2400	42.3%	-
20	9.5	3.5	8.5	8.0	20.0	2075	1825	113.7%	250
30	6.5	1.5	9.5	10.0	15.0	1755	2400	73.1%	-
40	12.0	16.0	25.0	30.0	0.0	2620	2400	109.2%	220
50	4.0	1.0	2.0	1.0	10.0	820	2400	34.2%	-
60	30.0	10.0	9.0	10.0	2.0	1680	2400	70.0%	-
Demand	20	30	40	30	60				
Sales price\$	30	50	50	40	20				
Material Price \$	10	42	25	25	15				

Table 2. Production Priority analysis

	A	B	C	D	E	Total Throughput
Throughput per product	20	8	25	15	5	
Time on constrained resource (20)	9.5	3.5	8.5	8	20	
Throughput unit per unit resource (20)	2.11	2.29	2.94	1.88	0.25	
Priority	3(20)	2(30)	1(40)	4(30)	5(48)	\$ 2325

In this paper we discussed the process of applying tabu-SA algorithm to solve the multi constraint TOC product mix decision problem.

III. PROPOSED ALGORITHM

a) Tabu-SA Algorithm

It has been seen that tabu search algorithm uses short term memory of recently visited solution known as tabu list to escape from local optima, but tabu list generated has a deterministic nature and thus cannot avoid cycling. This drawback of tabu list has been taken care by SA in tabu-SA algorithm due to its stochastic nature. But, it needs more iteration to converge at optimal or near optimal solution, because it does not keep any memory of recently visited solution and there is a chance for the search to return to visited solutions again. So with the help of short

term memory the search can be restricted from returning to a previously visited solutions and performance of SA can be enhanced significantly [7]. A hybrid algorithm named tabu-SA has been proposed in this paper. It covers a large number of solutions and the number of revisited is reduced with the help of tabu list. Such a hybrid algorithm for product mix problem is given in the next section.

b) Notations

- S : Current feasible product mix
- S_p : Product mix by perturbation of S and it is feasible one.
- S_b : Best product mix encountered so far.
- $TH(S)$: Total throughput of product mix S
- $TH(S_p)$: Throughput for product mix S_p
- ΔTH : Change in throughput due to perturbation over $TH(S)$
- ΔTH_b : Change in throughput due to perturbation over $TH(S_b)$
- $Reject$: Variable that stores the number of product mix rejected so far, when its value equal to iteration process is stopped.
- K : Number of iteration.
- R : Random number between 0 and 1
- T : Temperature
- A : Aspiration

c) Steps of proposed algorithm

Step 1- Using the TOC rule, evaluate the initial solution and assign it to S . Calculate its throughput value $TH(S)$

Step 2- Initialize $T=T_0=200$; $K=1$; $TL=\Phi$; $Reject=0$; $S_b=S$;

Step 3- generate (S_p, S)

that generate new solution from the existing one by randomly selecting two product type and replace it by two randomly generated number.

Step 4- see the feasibility of solution using equations (2) and (3). If S_p satisfies all the conditions go to next step else go to *Step 3* and generate a new solution.

Step 5- If $S_p \in TL$
Then go to *step 6*
Else go to *step 7*

Step 6- If $TH(S_p) \geq A$
Then go to *step 7*
Else go to *step 3*

Step 7- $\Delta TH = TH(S_p) - TH(S)$
If $\Delta TH \geq 0$
Then go to *step 8*
Else go to *step 9*

Step 8- Assign
 $S = S_p$
Include S_p in tabulist
 $TL \leftarrow S_p$
Update aspiration
 $A \leftarrow TH(S_p)$

Step 9- $\Delta TH_b = TH(S_p) - TH(S_b)$
If $\Delta TH_b \geq 0$
Then go to Step 10
Else go to Step 14

Step 10- Assign
 $S_b = S_p$
 $Reject \leftarrow 0$
go to Step 14

Step 11- Compute
 $P = \exp(\Delta TH / T)$
generate a random number R between (0,1)
if ($P \leq R$)
go to Step 13
else go to next step

Step 12- Assign
 $S = S_p$
Include S_p in tabu list
 $A \leftarrow TH(S_p)$

Step 13- $Reject = Reject + 1$;
If $Reject \geq 3$
go to Step 15
else go to Step 14

Step 14 - $K = K + 1$;
Change the temperature
 $T = T_0 / (1 + \ln K)$
If $K \geq 15$
go to Step 15
else go to Step 3

Step 15- FREEZE
 S_b is the near optimal solution.

IV. COMPUTATIONAL EXPERIENCE

An in-depth study is carried out to evaluate the computational performance of the tabu-SA based algorithm on multi constraints product mix decision problem. The main features of the tabu-SA algorithm are:

- (1) It is stochastic in nature. Thus, prevents the search process from being entrapped in local optima. Hence, the diversification of tabu search is sealed.
- (2) The deterministic nature of tabu search leads the avoidance of stumbling block problem (i.e. it effectively avoid the cycling of search path) by the probabilistic nature of the hybrid algorithm.
- (3) The computational time to compute the near optimal solution is significantly reduced by application of tabu-list.

Efficient performance of an algorithm is checked accordingly by the relation of an objective function, which is a logical combination of constraints and its throughput. To prove the superiority of proposed algorithm it is tested on well known data set of a product mix decision problem. Hybrid tabu-SA can runs with any random sequence of product mix, but it is found that if we started with some specific rule it conserve toward optimal solution quickly.

Result obtained by proposed tabu-SA algorithm on the problem simulated by Patterson's [8] is compared with all the previous applied methodologies and the summarized result is presented in figure 1.

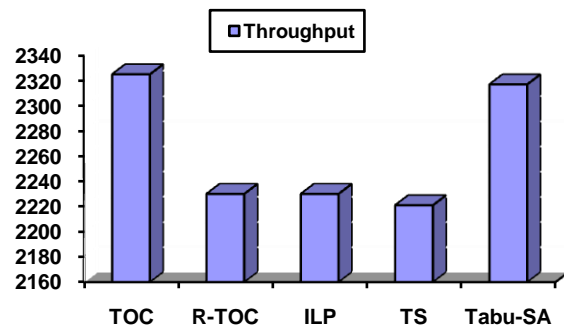


Figure 1. Throughput obtains from different heuristics.

The total throughput calculated by Fredendall and Lea [4] by TOC heuristic is \$2325. However, this product mix is unrealizable since one resource is overloaded, but when they solve that product mix problem by revised TOC they got total throughput is \$2230. This is also obtained by ILP method. But when same problem is solved by hybrid tabu-SA algorithm result comes out to be {A20, B30, C40, D29, E47} with throughput \$2317 without overloading of any resource. It reveal the better performance of proposed heuristic in compare to other existing one, and for most of the problem the proposed tabu-SA algorithm showing consistent performance. In this article algorithm target a specific type of product mix problem. Setting the value of T_0 , $Reject$, TL , and maximum number of iteration to certain values makes the algorithm perform better in this scenario. However, these variables have to be set to other value for different product mix problem of different size and types.

V. CONCLUSION

In this article a problem pertaining to Theory of constraints product mix decision has been addressed using a hybrid optimization approach called tabu-simulated annealing. The performance of tabu-SA heuristic has been tested on the problem reported in literature; found exemplary when the same has been compared with TOC based heuristic, revised-TOC, Tabu search based, and ILP. Tuning of various parameters of tabu-SA heuristic has been rigorously carried out and appropriate values have been selected after large number of trial runs.

In this paper attempt has been made to highlight the importance of product mix component in theory of constraint which is the building blocks of master production schedule to maximize the system throughput. In future several practical case studies with large data set will be addressed by other nature inspired algorithm like particle swarm optimization, ant colony, psycho-clonal algorithm etc. and the outcome will compared with the hybrid tabu-SA algorithm.

VI. REFERENCES

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